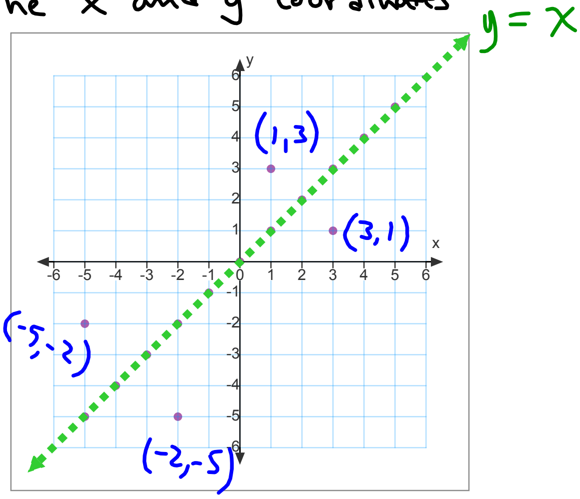
1.4 INVERSE of a Relation

Reflection in the line $y = x$ switches the x and y coordinates

$(x, y) \rightarrow (y, x)$



The Domain of the original function becomes the Range of the "new" Relation and vice versa

If the original function is $f(x)$, then the Inverse, if it is a function, is denoted $f^{-1}(x)$

In order for a function to have an inverse that is also a function it MUST pass a horizontal line test.

If a function passes the HLT, we say its inverse exists, meaning its inverse is a function. These functions are called one-to-one functions.

Example: Given $f(x) = 2x+1$

Domain: $\{x \mid x \in \mathbb{R}\}$

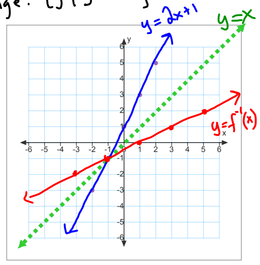
Range: $\{y \mid y \in \mathbb{R}\}$

$y=2x+1$

x	y
-2	-3
-1	-1
0	1
1	3
2	5

INVERSE

x	y
-3	-2
-1	-1
1	0
3	1
5	2



To get $f^{-1}(x)$ rewrite as $y=2x+1$ then switch x and y in the equation and then rearrange to get y by itself.

Given $y = 2x+1$

Inverse is $x = 2y+1$

$x-1 = 2y$

$2y = x-1$

$y = \frac{x-1}{2}$

$\therefore f^{-1}(x) = \frac{x-1}{2}$

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \in \mathbb{R}\}$

Example 2: Given $y = x^2+1$

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 1, y \in \mathbb{R}\}$

what is the inverse of $y = x^2+1$?

INVERSE: $x = y^2+1$

$x-1 = y^2$

$y^2 = x-1$

$y = \pm\sqrt{x-1}$

D: $\{x \mid x \geq 1, x \in \mathbb{R}\}$

R: $\{y \mid y \in \mathbb{R}\}$

Is this a function? No!

How can we restrict the domain of $y=x^2+1$ so that its inverse is a function?

Solution: cut it at the vertex

vertex is $(0,1)$

Either say $x \geq 0$ or $x \leq 0$

Now our function is $f(x) = x^2+1, x \leq 0$

D: $\{x \mid x \leq 0, x \in \mathbb{R}\}$

R: $\{y \mid y \geq 1, y \in \mathbb{R}\}$

From our work earlier we got the inverse to be $y = \pm\sqrt{x-1}$

Base on the restriction we chose what is $f^{-1}(x)$?

$f^{-1}(x) = -\sqrt{x-1}$

D: $\{x \mid x \geq 1, x \in \mathbb{R}\}$

R: $\{y \mid y \leq 0, y \in \mathbb{R}\}$

Other questions on inverses (Examples)

1) If $f(3)=5$, what is $f^{-1}(5)$?

$$f^{-1}(5)=3$$

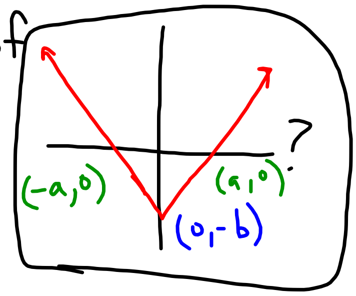
(b) what is $f^{-1}(f(3))=3$

$x \to y \to x$

$3 \to 5 \to 3$

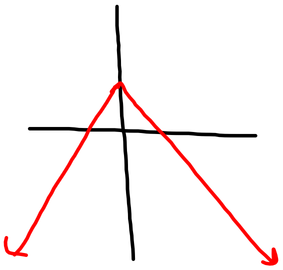
$$f(f^{-1}(f(3)))=5$$

2) which is the inverse of

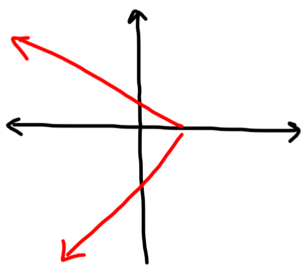


?

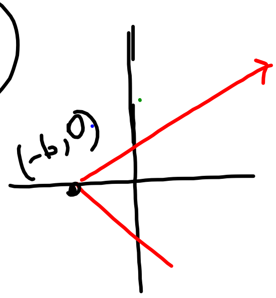
(a)



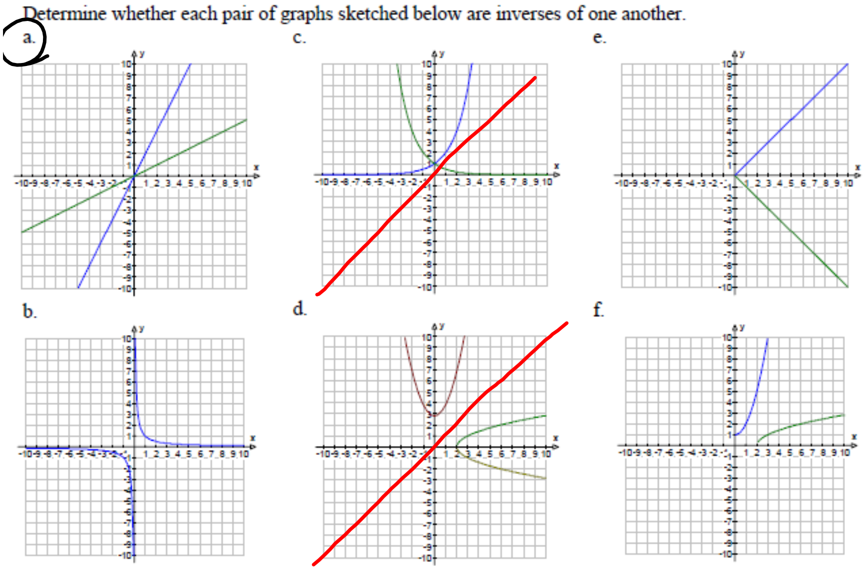
(b)



(c)



Determine whether each pair of graphs sketched below are inverses of one another.



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