4.5 Equations/word problems with Rational Expressions

Ex 1

cost = \$100

	cost	# in group	cost per person (Each)	
original	100	x	$\frac{100}{x}$	less
new group	100	$x-5$	$\frac{100}{x-5}$	more

more − less = difference

$$\left[\frac{100}{x-5} - \frac{100}{x} = 1\right]$$

no factors in common in denominators ∴ LCD is the product of the denominators

LCD = $x(x-5)$

$$\cancel{x(x-5)}\frac{100}{\cancel{x-5}} - \cancel{x}(x-5)\frac{100}{\cancel{x}} = x(x-5)(1)$$

$$100x - 100(x-5) = x(x-5)$$

$$\cancel{100x} - \cancel{100x} + 500 = x^2 - 5x$$

$$0 = x^2 - 5x - 500$$

$$0 = (x-25)(x+20)$$

$$x - 25 = 0 \quad \text{OR} \quad x + 20 = 0$$

$$x = 25 \quad \text{OR} \quad \cancel{x = -20}$$

So, there were 25 students in the original group.

Ex①: The sum of a number and its reciprocal is $\frac{5}{2}$, what is the number?

Solution: Let x be the number

Then its reciprocal is $\frac{1}{x}$

Equation is: $\boxed{x + \frac{1}{x} = \frac{5}{2}}$

Ex 2

	km distance	km/h speed	h time $\left(\frac{d}{s}\right)$
car	270	$x+10$	$\frac{270}{x+10}$
bus	160	x	$\frac{160}{x}$

(time in car) + (time on bus) = 5

$$\frac{270}{x+10} + \frac{160}{x} = 5$$

LCD: $x(x+10)$

$$\cancel{x(x+10)}\frac{270}{\cancel{x+10}} + \cancel{x}(x+10)\frac{160}{\cancel{x}} = 5x(x+10)$$

$$270x + 160(x+10) = 5x(x+10)$$

$$270x + 160x + 1600 = 5x^2 + 50x$$

$$430x + 1600 = 5x^2 + 50x$$

$$0 = 5x^2 + 50x - 430x - 1600$$

$$\frac{0 = 5x^2 - 380x - 1600}{5}$$

$$0 = x^2 - 76x - 320$$

$$0 = (x-80)(x+4)$$

$$x - 80 = 0 \quad \text{OR} \quad x + 4 = 0$$

$$x = 80 \quad \text{OR} \quad \cancel{x = -4}$$

So bus is travelling at 80 km/h and car is travelling at 90 km/h

Ex 3

Together it takes 2.4 hrs

In one hr it fills $\frac{1}{2.4}$

Assume it takes x hrs for pump B to fill the tank

In one hr it fills $\frac{1}{x}$ of the tank

Pump A will take $x+2$ hrs and fill $\frac{1}{x+2}$ in one hr

In one hr: pump A + pump B = together

$$2.4x(x+2)\left(\frac{1}{x+2}+\frac{1}{x}=\frac{1}{2.4}\right)$$

$$2.4x\cancel{(x+2)}\frac{1}{\cancel{x+2}}+2.4\cancel{x}(x+2)\frac{1}{\cancel{x}}=\cancel{2.4}x(x+2)\frac{1}{\cancel{2.4}}$$

$$2.4x+2.4(x+2)=x(x+2)$$

$$2.4x+2.4x+4.8=x^2+2x$$

$$4.8x+4.8=x^2+2x$$

$$0=x^2+2x-4.8x-4.8$$

$$0=x^2-2.8x-4.8$$

$$0=(x-4)(x+1.2)$$

$x-4=0$ OR $x+1.2=0$

$\boxed{x=4}$ OR $\cancel{x=-1.2}$ ☹

NO GOOD

$10x^2-28x-48$

$5x^2-14x-24$

OR use

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

It takes pump B 4 hrs and it take pump A 6 hrs