7.2 Evaluating Log Expressions

Note: Every log expression can be written as an exponential and vice versa.

$$\log_b N = E \Leftrightarrow b^E = N$$

Ex: $\log_2 8 = 3 \Rightarrow 2^3 = 8$

$\log_5 625 = 4 \Rightarrow 5^4 = 625$

$2^6 = 64 \Rightarrow \log_2 64 = 6$ Exponent

$9^{\frac{1}{2}} = 3 \Rightarrow \log_9 3 = \frac{1}{2}$

Evaluating logs: Think "what exponent do we need to put on the base to get the number"?

Ex: ① $\log_3 9 = 2$ because $3^{\boxed{2}} = 9$

we could also let the log expression equal some variable, like x, and switch to, and solve, the corresponding exponential equation.

Ex: ② Evaluate: $\log_2 \frac{1}{16}$

let $\log_2 \frac{1}{16} = x$

then switch $\Rightarrow 2^x = \frac{1}{16}$

$2^x = \frac{1}{2^4}$

$\cancel{2}^x = \cancel{2}^{-4}$

$\boxed{x = -4}$

$\therefore \log_2 \frac{1}{16} = -4$

Ex: ③ Evaluate $\log_{27} 3$

let $\log_{27} 3 = x$

switch $27^x = 3$

$(3^3)^x = 3^1$

$3^{3x} = 3^1$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{1}{3}$

$\boxed{x = \frac{1}{3}}$

So $\log_{27} 3 = \frac{1}{3}$

Ex ④ Evaluate: $\ln 8$ (use calculator)

$\ln 8 = 2.079$ $\left[e^{2.079} = 8\right]$

Ex ⑤ Evaluate $\log 0.001$

$\log 0.001 = x$

$10^x = 0.001$

$10^x = \frac{1}{1000}$

$10^x = 10^{-3}$

$x = -3$

so

$\log 0.001 = -3$

page 436 # 1-15

Solutions to some questions

(b) $pH = -\log x$

$2 = -\log x$

$-2 = \log x$

$10^{-2} = x$

$x = 0.01$ mol/L

(c) $1.6 = -\log x$

$-1.6 = \log x$

$x = 10^{-1.6}$

$x = 0.02512$

$2.5 = -\log x$

$-2.5 = \log x$

$x = 10^{-2.5}$

$x = 0.003162$

$\frac{0.02512}{0.003162} = 7.9$ times more acidic