7.4 Solving Exp. equations using logs.

Ex: Solve for x : $2^x = 7$

Take log (base 10) of both sides

$$\log 2^x = \log 7$$

$$\frac{x \cdot \cancel{\log 2}}{\cancel{\log 2}} = \frac{\log 7}{\log 2}$$

$$x = \log(7) \div \log(2)$$

$$x = 2.81$$

This process will also allow us to evaluate any log

Ex: Evaluate $\log_3 15$

let $\log_3 15 = x$

switch to exponential form

$$3^x = 15$$

$$\log 3^x = \log 15$$

$$\frac{x \cancel{\log 3}}{\cancel{\log 3}} = \boxed{\frac{\log 15}{\log 3}}$$

$$x = 2.46$$

$$\boxed{\log_3 15 = \frac{\log 15}{\log 3}}$$

In fact,

$$\log_b N = \frac{\log_a N}{\log_a b}$$

where a is any base we choose.

$$\boxed{\text{property: } \log_b a = \frac{\log_c a}{\log_c b}}$$

choose base 10 for calculator use!

Ex: Evaluate $\log_5 10 = \frac{\log 10}{\log 5} = 1.43$

Ex: solve for x: $3^{x-1} = 20$

$$\log 3^{(x-1)} = \log 20$$

$$\frac{(x-1)\cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 20}{\log 3}$$

$$x-1 = \frac{\log 20}{\log 3}$$

$$x = \frac{\log 20}{\log 3} + 1$$

$$x = 3.7268$$

Ex: $2^{3x+1} = 5^{2x-1}$

$$\log 2^{(3x+1)} = \log 5^{(2x-1)}$$

$$(3x+1)\log 2 = (2x-1)\log 5$$

$$(3x+1)0.301 = (2x-1)0.699$$

$$0.903x + 0.301 = 1.398x - 0.699$$

$$0.301 + 0.699 = 1.398x - 0.903x$$

$$\frac{1}{0.495} = \frac{0.495x}{0.495}$$

$$\boxed{2.02 = x}$$

Problem solving with Logs

Ex: The half-life of a certain element is 520 yrs. How long will it take for 50g to decay to 5g?

Solution: $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{520}}$

$$\frac{5}{50} = \frac{\cancel{50}}{\cancel{50}}\left(\frac{1}{2}\right)^{\frac{t}{520}}$$

$$\frac{1}{10} = \left(\frac{1}{2}\right)^{\frac{t}{520}}$$

$$\log 0.1 = \log(0.5)^{\frac{t}{520}}$$

$$\log 0.1 = \frac{t}{520}\log(0.5)$$

$$\frac{-1}{-0.301} = \frac{t}{520}\frac{(\cancel{-0.301})}{\cancel{-0.301}}$$

$$\frac{3.32}{1} = \frac{t}{520}$$

$$t = (3.32)(520)$$

$$\boxed{t = 1726.4\text{ yrs}}$$

p. 455

1-13, 15, 16

Ex: $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{3}}$

15% remaining $\Rightarrow \frac{A}{A_0} = 0.15$

$$\frac{15}{100} = \frac{100\left(\frac{1}{2}\right)^{\frac{t}{3}}}{100}$$

$$= \left(\frac{1}{2}\right)^{\frac{t}{3}}$$

$$0.15 = \left(\frac{1}{2}\right)^{\frac{t}{3}}$$

$$\log 0.15 = \log\left(\frac{1}{2}\right)^{\frac{t}{3}}$$

$$\log 0.15 = \frac{t}{3}\log\left(\frac{1}{2}\right)$$

$$\left[-0.8239 = \frac{t}{3}(-0.301)\right] \times 3$$

$$3(-0.8239) = t(-0.301)$$

$$\frac{-2.4717}{-0.301} = \frac{t\cancel{(-0.301)}}{\cancel{-0.301}}$$

$$\boxed{8.2 = t}$$

p. 458
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\$2000 in debt, 18.5% annual rate compounded monthly

double debt = \$4000

$$A = P(1+i)^n$$

$$A = 2000\left(1+\frac{0.185}{12}\right)^{12t}$$, t is years

$$\frac{4000}{2000} = \frac{\cancel{2000}(1.0154)^{12t}}{\cancel{2000}}$$

$$2 = (1.0154)^{12t}$$

$$\log 2 = \log(1.0154)^{12t}$$

$$\frac{\log 2}{12\log(1.0154)} = \frac{\cancel{12}t\,\cancel{\log(1.0154)}}{\cancel{12}\,\cancel{\log(1.0154)}}$$

$$3.78 = t$$

It takes 3.78 yrs

Worksheet

#11. $p(x) = -\log x$ ($p(x)$ = pH level)

(A) $x = 0.0002$ mol/L

$pH = p(0.0002) = -\log(0.0002)$

$pH = p(0.0002) = -(-3.69897)$

$pH = 3.7$

(B) $pH = 6.6 = p(x)$

$p(x) = -\log x$

$6.6 = -\log x$

$-6.6 = \log x$

$10^{-6.6} = x$

$x = 2.5 \times 10^{-7}$

$x = 0.00000025$ mol/L

(C) $pH = 1.8$ compared to $pH = 2.4$

$1.8 = -\log x$

$x = 10^{-1.8}$

0.01585

$2.4 = -\log x$

$x = 10^{-2.4}$

0.00398

$0.01585 - 0.00398$

$= 0.01187$ mol/L