Writing Quad. Equations from Roots, zeros or x-intercepts

Solve: To get Roots

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$x+1=0 \quad x-5=0$

$x=-1 \quad x=5$

Use Roots to write factors

If $x=-1$ and $x=5$

then $(x+1)$ and $(x-5)$ are the factors

We could also say that if $x=-1$ and $x=5$

then $x+1=0$ and $x-5=0$

therefore $(x+1)(x-5)=0$

This is a quadratic equation with roots $x=-1$ and $x=5$

The quadratic $(x+1)(x-5)=0$ has roots $x=-1$ and $x=5$ so too does:

$$3(x+1)(x-5)=0$$
$$4(x+1)(x-5)=0$$
$$-2(x+1)(x-5)=0$$

In facts any quadratic of the form $a(x+1)(x-5)=0$ where a is a constant (Except $a \neq 0$) will have Roots $x=-1$ and $x=5$

Ex: (a) Roots -8 and 3 and $a=1$

$(x+8)(x-3)=0$ factored form

$x^2 - 3x + 8x - 24 = 0$

$x^2 + 5x - 24 = 0$ standard form

(c) Roots are 0 and -5 and $a=1$

$(x+0)(x+5)=0$ factored form

OR $x(x+5)=0$ factored form

$x^2+5x=0$ standard form

Ex: Roots are $\frac{1}{2}$ and -3 and $a = 1$

$\boxed{x = \frac{1}{2}} \cdot 2$

$(2x-1)(x+3) = 0$

$2x - 1 = 1 - 1$

$(2x-1) = 0$

From x-intercepts:

Ex: x-intercepts are -3 and 4

$$a(x+3)(x-4)=0$$

what if this graph also has a y-intercept at $(0,-5)$? How can we use this to get the exact equation (function), ie. what is a?

x y

The equation $a(x+3)(x-4)=0$ came from the function $a(x+3)(x-4)=y$

Now we can sub in the (x,y) from $(0,-5)$ to solve for a

So we get $a(0+3)(0-4)=-5$

$$a(3)(-4)=-5$$

$$\frac{a\cancel{(-12)}}{\cancel{-12}}=\frac{-5}{-12} \Rightarrow a=\frac{5}{12}$$

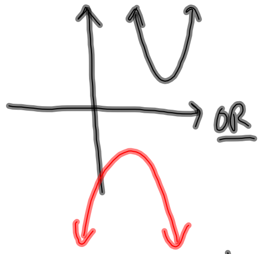
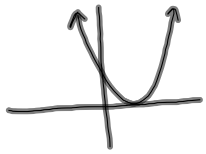
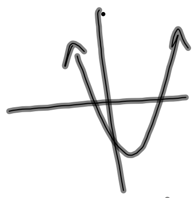
Then the equation can be written as

$$\frac{5}{12}(x+3)(x-4)=0$$

and the function is $y=\frac{5}{12}(x+3)(x-4)$

Quadratic formula and the nature of the Roots

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$b^2-4ac < 0$ negative	$b^2-4ac = 0$	$b^2-4ac > 0$ positive
NO REAL SOLUTIONS	REAL EQUAL ROOTS	REAL UNEQUAL ROOTS
		
No x-intercepts	one x-intercept at vertex	two x-intercepts
		b^2-4ac is a perfect square then roots are rational
		b^2-4ac not a perfect square Roots are irrational

Ex: Describe the Roots of

$$3x^2-2x+4=0$$

$a=3 \quad b=-2 \quad c=4$

$$b^2-4ac = (-2)^2-4(3)(4)$$
$$4-48$$
$$-44$$

No REAL ROOTS

Ex: $2x^2+5x-3=0$

$$b^2-4ac$$
$$5^2-4(2)(-3)$$
$$25+24$$
$$49$$

since $49>0$ Roots are Real and unequal

also since 49 is a perfect square, Roots are Rational.

UNIT 7 Summary:

Solving Quadratic Equations by:

- factoring and ZPP $a \cdot b = 0$, then $a = 0$ or $b = 0$
- quadratic formula
- graphing
- square root both sides

Connection between Roots, zeros and x-intercepts

Solving problems with quadratics

writing equations from roots/zeros/x-intercepts

$b^2 - 4ac$ and the nature of the roots.