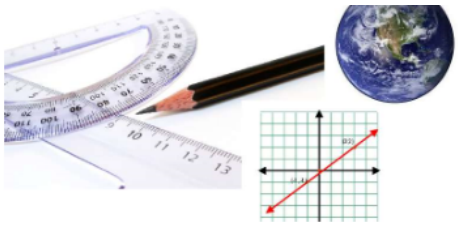
Proportional Reasoning
Lesson #1: Review



Conversion of Units

Metric System of Measurement with the Relationship to 1 Metre, 1 Gram, or 1 Litre

Prefix	**giga**	**mega**	**kilo**	**hecto**	**deca**		**deci**	**centi**	**milli**	**micro**	**nano**
Symbol	**G**	**M**	**k**	**h**	**da**		**d**	**c**	**m**	**μ**	**n**
Units	1 000 000 000	1 000 000	1 000	100	10	1	0.1	0.01	0.001	0.000 001	0.000 000 001
Symbol	Gm Gg GL	Mm Mg ML	km kg kL	hm hg hL	dam dag daL	*m* *g* *L*	dm dg dL	cm cg cL	mm mg mL	μm μg μL	nm ng nL

Metric Unit Number Line

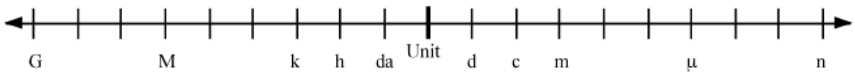
Each movement of one position represents a power of 10.

Conversion Within and Between the SI and Imperial Systems

Imperial to Imperial	Imperial to SI (Metric)	SI (Metric) to Imperial
1 foot (ft) = 12 inches (in)	1 in = 2.54 cm	1 cm = 0.3937 in
1 yard (yd) = 3 feet (ft)	1 ft = 0.3048 m	1 m = 3.2808 ft
1 mile (mi) = 5280 ft	1 yd = 0.9144 m	1 m = 1.0936 yds
1 mi = 1760 yards (yds)	1 mi = 1.6093 km	1 km = 0.6214 mi

Math 2201 Proportional Reasoning Unit 8 Lesson 1

Using the Metric Unit Number Line to Convert Within SI Units



Use the metric unit number line to complete the following.

a) 1.4 m to cm **b)** 2.23 L to mL **c)** 652 cm to m **d)** 42 480 mg to kg

1.4 x 100 = 140

2.23 x 1000 = 2230 mL

652 ÷ 100 = 6.52 m

42 480 ÷ 1 000 000 = 0.042 480 kg

Using Proportions to Convert

Require
i) a commonly known conversion.
or
ii) a ratio given in the question.

ratio $\frac{a}{b} = \frac{c}{d}$ **It is essential that the units are equivalent in each ratio.**

What is the actual height of a building if a model measures 10.5 cm?
Considering the scale 1 cm represents 2.3 feet.

1 cm = 2.3 ft

10.5 cm = x 2.3

Use proportional reasoning to convert 1.05 miles to inches.

Math 2201 Proportional Reasoning Unit 8 Lesson 1

Unit Analysis to Convert

In unit analysis, if only one conversion is required, the basic set-up in this approach is with units being placed properly in the numerator and denominator.

Convert 3.8 km to cm.

$3.8\,\text{km} \times \frac{1000\,\text{m}}{1\,\text{km}} = 3800\,\text{m} \times \frac{100\,\text{cm}}{1\,\text{m}} = 380\,000\,\text{cm}$

Convert 1.4km to mm.

Convert 0. 15 miles to inches

$0.15\,\text{mi} \times \frac{5280\,\text{ft}}{\text{mi}} = 792\,\text{ft} \times \frac{12\,\text{in}}{\text{ft}} = 9504\,\text{in}$

Convert an answer of 34 m/s to km/min.

Math 2201 Proportional Reasoning Unit 8 Lesson 1

Slope Review

In this unit, we also use the concept of **slope** (the steepness of a line) as a rate. We will review basic concepts of slope. Recall the following:

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$\frac{\Delta y}{\Delta x}$

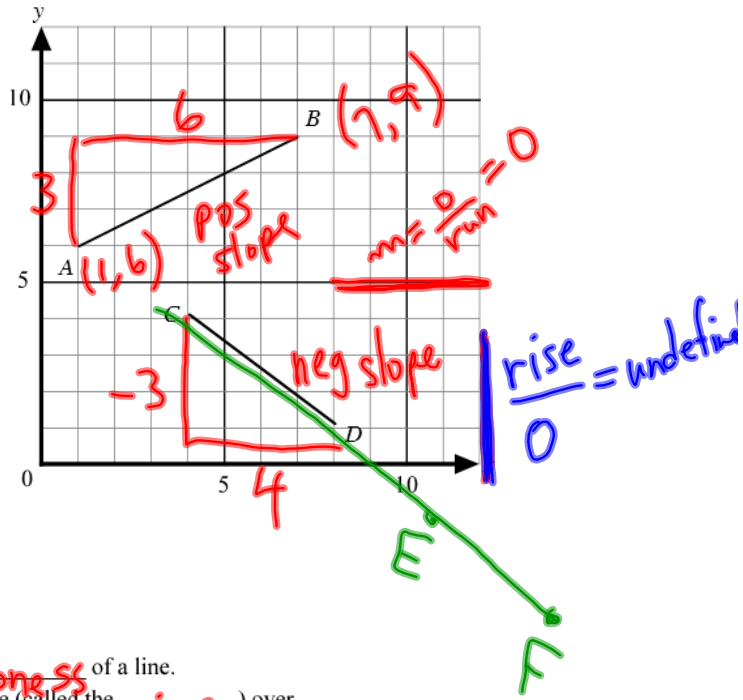
Consider line segments *AB* and *CD* shown.

a) Determine the slope of each line by using $m = \frac{\text{rise}}{\text{run}}$.

$m_{AB} = \frac{3}{6} = \frac{1}{2}$

b) Determine the slopes of each line using $m = \frac{y_2 - y_1}{x_2 - x_1}$.

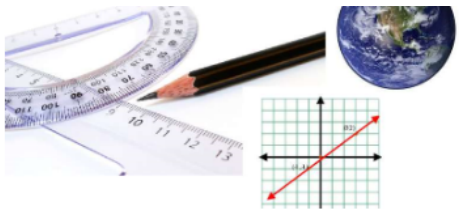
$m = \frac{9-6}{7-1} = \frac{3}{6} = \frac{1}{2}$



Slope Facts

Slope is the measure of the steepness of a line.
Slope is the ratio of the vertical change (called the rise) over the horizontal change (called the run).
A line segment which rises from left to right has a positive slope.
A line segment which falls from left to right has a negative slope.
A horizontal line segment has a slope of 0.
A vertical line segment has an undefined slope.
The slopes of all line segments on a line are equal.

Proportional Reasoning
Lesson #2: Rates and Ratios



Rates and Ratios

- Whereas a **ratio** compares quantities with the same units, a **rate** compares quantities with different units.
- Typically a **ratio** compares quantities of the same type of object so no unit is included in the calculation.
- Unless stated otherwise, **ratios** are always simplified and may be written in the form $a:b$ or $\frac{a}{b}$.
- A **rate** is a specific type of ratio. A rate is a comparison between 2 quantities where one is changing relative to the other with different measuring units for the two quantities.
- A unit rate is a rate expressed with a denominator of 1. For exampe, $\frac{15\text{ m}}{2\text{ s}}$ can be expressed as a unit rate of 7.5 m/s .

Rick hosted an annual lobster feast. Last year he bought 20 kg of lobster for $259.60.
This year, he bought the lobster for $6.80/lb. Note: 1kg = 2.2 lb.
Use unit analysis in your work to determine which lobster was more expensive.

$\frac{\$6.80}{1\text{ lb}} \times \frac{2.2\text{ lb}}{1\text{ kg}} = \$14.96/\text{kg}$

$\frac{\$259.60}{20\text{ kg}} = \$12.98/\text{kg}$

Math 2201 Proportional Reasoning Unit 8 Lesson 2

Property owners are required to pay property tax on an annual basis. The property tax amount is based on the mill rate and the assessed value of the property.

To calculate the property taxes, some municipalities use the formula

$$\text{Property Tax} = \frac{\text{Assessed Value}}{1000} \times \text{mill rate}.$$

a) If Todd paid $3 654.42 in property tax last year when the mill rate was 7.4026, what is his home's assessed value, to the nearest dollar?

$$\frac{3654.42}{7.4026} = \frac{A}{1000} \times \frac{7.4026}{7.4026}$$

$$1000(493.667) = \frac{A}{1000} \cdot 1000$$

$A = \$493\,677$

b) How much does he property tax bill change for the current year if his property is assessed 10% higher and the mill rate decreases to 6.7221?

$$493\,667 \times 1.10 = \$543\,034$$

$$\text{Tax} = \frac{543\,034}{1000} \times 6.7221 = \$3650.34$$

Math 2201 Proportional Reasoning Unit 8 Lesson 2

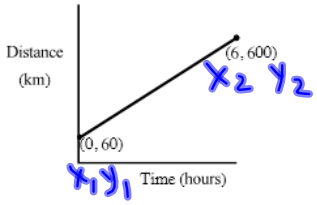
Slope as a Rate of Change

Slope represents a **rate of change**.

- Rate of change can be used to describe many scenarios, ie change in distance travelled to change in time, change in temperature to change in time, etc.
- A **positive slope** indicates a **positive rate of change**.
- A **negative slope** indicates a **negative rate of change.**

Steve is taking a road trip.

The graph shown represents the distance travelled over time in hours.



a) What does the point (0, 60) on the graph represent?

start

b) What does the point (6, 600) on the graph represent?

600 km at 6 hrs

c) Calculate the slope of the line joining the two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{600 - 60}{6 - 0} = \frac{540}{6} = 90$$

d) The slope represents a rate of change - a change in distance divided by a change in time. What units are used to represent this rate of change?

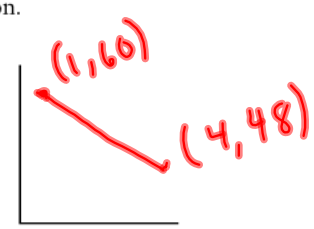
90 km/h

e) Complete:
The distance is (increasing/decreasing) at the rate of 90 km per hr.

Math 2201 Proportional Reasoning Unit 8 Lesson 2

Oil is leaking out of the bottom of a storage container at a constant rate. After 1 day the oil level is 60 inches and after 4 days the oil level is 48 inches.

a) On the grid, plot the ordered pairs to represent the given information.



b) Calculate the slope of the line segment joining the ordered pairs.

$$m = \frac{48-60}{4-1} = -\frac{12}{3} = -\frac{4}{1}$$

c) The slope represents a **rate of change** - a change in oil level divided by a change in time. What units are used to represent this rate of change?

$$-4 \frac{\text{inches}}{\text{day}}$$

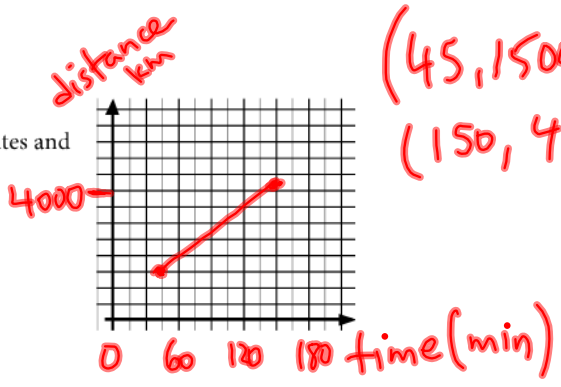
d) Complete the following to explain what the rate of change in this scenario represents.

The oil level is changing/decreasing at the rate of 4 inches per day.

A plane travelled 1500 km in 45 minutes. After 2 hours and 30 minutes the plane travelled 4200 km.

a) On the grid, plot the ordered pairs to represent the given information.

b) Calculate the planes average speed between 45 minutes and 2hours and 30 minutes.



(45, 1500)
(150, 4200)

$$m = \frac{4200-1500}{150-45}$$

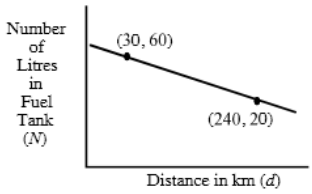
$$m = \frac{2700}{105} = 25.7 \text{ km/min}$$

$$25.7 \frac{\text{km}}{\text{m}} \times \frac{60 \text{ m}}{1 \text{ h}}$$

1542 km/h

Math 2201 Proportional Reasoning Unit 8 Lesson 2

The graph shown represents the amount of fuel in a gas tank as a function of the distance travelled by a car travelling at a constant speed.



a) Calculate the slope of the line.

$m = \frac{-40}{210} = \frac{-4}{21} = -0.19$

b) Complete the following statements.

The amount of fuel in the tank is (increasing/decreasing).

The rate of change of fuel in the fuel tank is -0.19 L per km.

The amount of fuel in the fuel tank is decreasing at the rate of 0.19 L per km

$\frac{0.19\text{ L}}{1\text{ km}}\left(\frac{100}{100}\right) = \frac{19\text{ L}}{100\text{ km}}$

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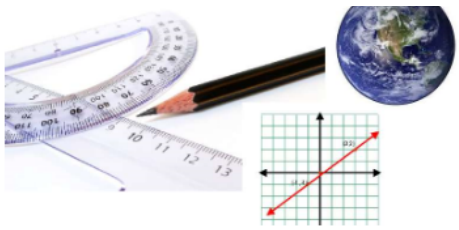
Proportional Reasoning
Lesson #3
Linear Scale Factors and Perimeter

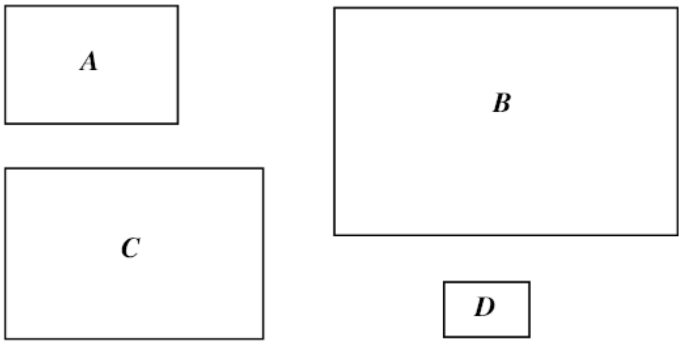


A linear scale factor describes the enlargement or reduction of length. It is described as a ratio in the form $a : b$ or as a rational number $\frac{a}{b}$, or as a percent. For example

- The **linear scale factor** of $\square B$ from $\square A$ is $2 : 1$ or 2 or 200%
- The **linear scale factor** of $\square D$ from $\square A$ is $1 : 2$ or 0.5 or 50%

A scale factor **greater than 1** describes an **enlargement**.
A scale factor **between 0 and 1** describes a **reduction**.

Rectangles B, C, and D are all similar to rectangle A.



1. Measure in cm, and then complete the following.

a) i) $\frac{\text{Length of } \square B}{\text{Length of } \square A} =$ **ii)** $\frac{\text{Width of } \square B}{\text{Width of } \square A} =$

The length of B is _____ times the length of A. The width of B is _____ times the width of A.

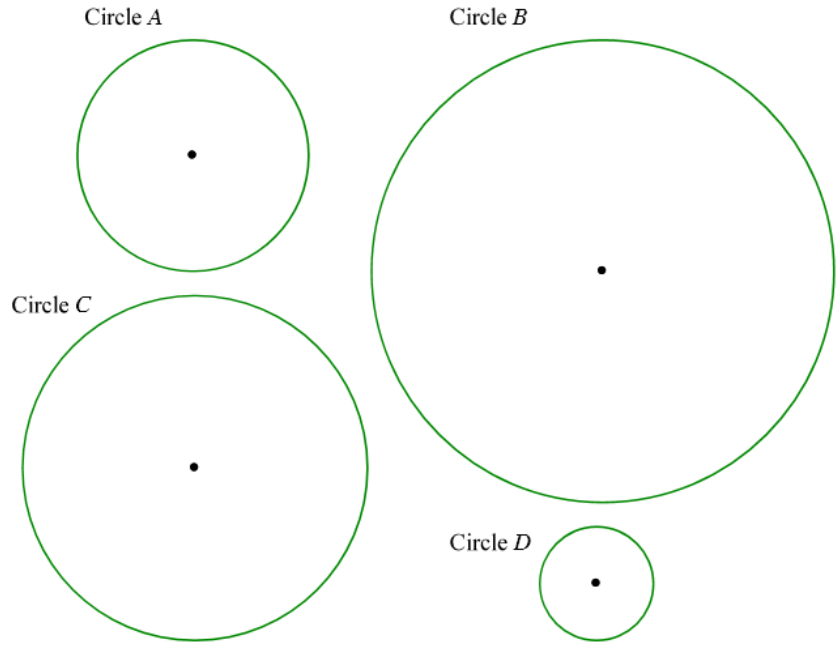
b) i) $\frac{\text{Length of } \square C}{\text{Length of } \square A} =$ **ii)** $\frac{\text{Width of } \square C}{\text{Width of } \square A} =$

The length of C is _____ times the length of A. The width of C is _____ times the width of A.

c) i) $\frac{\text{Length of } \square D}{\text{Length of } \square A} =$ **ii)** $\frac{\text{Width of } \square D}{\text{Width of } \square A} =$

The length of D is _____ times the length of A. The width of D is _____ times the width of A.

Math 2201 Proportional Reasoning Unit 8 Lesson 3



a) Measure, to the nearest cm, the radius of each circle and write the measure on the diagram.

b) Circles *B*, *C*, and *D* are enlargements or reductions of Circle *A*. Determine the linear scale factor in each case.

- The **linear scale factor** of circle *B* from circle *A* is
- The **linear scale factor** of circle *C* from circle *A* is
- The **linear scale factor** of circle *D* from circle *A* is

Math 2201 Proportional Reasoning Unit 8 Lesson 3

c) Calculate the circumference of each circle as an exact value (i.e. as a multiple of π).
$C = 2\pi r$.

d) Determine the perimeter scale factors by completing the following.

i) Perimeter scale factor of B from $A = \frac{\text{Circumference of } B}{\text{Circumference of } A} =$

ii) Perimeter scale factor of C from $A = \frac{\text{Circumference of } C}{\text{Circumference of } A} =$

iii) Perimeter scale factor of D from $A = \frac{\text{Circumference of D}}{\text{Circumference of A}} =$

e) What do you notice about the linear scale factors and the perimeter (circumference) scale factors?

a) Complete the following.

	Original Dimensions of Rectangle (cm)	Original Perimeter (cm)	Linear Scale Factor Applied to Rectangle	New Dimensions of Rectangle (cm)	New Perimeter (cm)	Perimeter Scale Factor $\frac{\text{New Perimeter}}{\text{Original Perimeter}}$
A	4 x 5		2 : 1 or 2			
B	2 x 6		3 : 1 *or* 3			
C	9 x 6		1 : 3 *or* $\frac{1}{3}$			
D	3 x 12		2 : 3 *or* $\frac{2}{3}$			

b) Compare the linear scale factors to the perimeter scale factors.

Perimeter scale factor linear scale factor

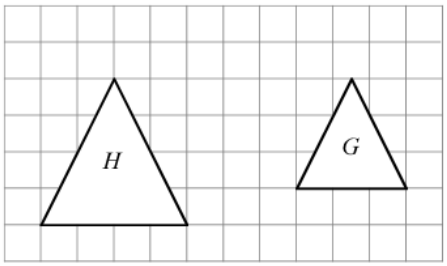
Math 2201 Proportional Reasoning Unit 8 Lesson 3

Determine the scale factor that will tranform figure H to figure G

a) a ratio

b) a rational number

c) a percent



Jonathan increased the length and width of a rectangular 8″ x 12″ poster by a factor of 7:3.

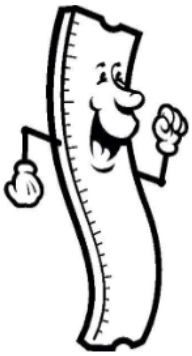
a) Is this an enlargement or reduction?

b) Calculate the new dimensions of the picture.

c) Show that the ratio $\frac{\text{new perimeter}}{\text{original perimeter}} = \frac{7}{3}$

Proportional Reasoning

Lesson #4: Linear Scale Factors and Area



a) Complete the following table.

	Original Dimensions of Rectangle (cm)	Original Area (cm²)	Linear Scale Factor Applied to Rectangle	New Dimensions of Rectangle (cm)	New Area (cm²)	Area Scale Factor New Area: Original Area $\frac{\text{New Area}}{\text{Original Area}}$
A	3 x 5		2 : 1 *or* 2			$\frac{\text{New Area}}{\text{Original Area}} = \quad \Rightarrow$
B	5 x 6		1 : 4 *or* $\frac{1}{4}$			$\frac{\text{New Area}}{\text{Original Area}} = \quad \Rightarrow$
C	4 x 12		5 : 3 *or* $\frac{5}{3}$			$\frac{\text{New Area}}{\text{Original Area}} = \quad \Rightarrow$
D	6 x 8		2 : 3 *or* $\frac{2}{3}$			$\frac{\text{New Area}}{\text{Original Area}} = \quad \Rightarrow$

b) Complete the following statement:

area scale factor = (linear scale factor)

Dione has a 5" x 6" childhood photograph on her laptop.

a) Dione would like to increase the area by 55%.
Determine the area scale factor and the linear scale factor.

b) Explain why these scale factors are different.

c) Determine the dimensions of the enlarged photograph.

Math 2201 Proportional Reasoning Unit 8 Lesson 4

e) Dione must also produce a print whose area will be reduced by 25%. Determine the dimensions of the print to the nearest hundredth of an inch.

The area of the province of Newfoundland is approximately 405 212 km^2.

On tourist map, the area of Newfoundland is represented as 162.08 cm^2. The scale of the map (linear scale factor) can be written in the form $1 : x$. Calculate the value of x to the nearest whole number.

On the same map, the area of the province of Nova Scotia is repsented as 22.114 cm^2. Determine the actual area of Nova Scotia to the nearest square kilometre.

Proportional Reasoning
Lesson #5:
Linear Scale Factors, Surface Area, and Volume

Complete the following surface area scale factor for the following rectangular prisms.

	Original Dimensions (cm)	Original Surface Area (cm^2)	Linear Scale Factor	New Dimensions (cm)	New Surface Area (cm^2)	Surface Area Scale Factor New Surface Area : Original Surface Area $\frac{\text{New Surface Area}}{\text{Original Surface Area}}$
A	4 x 6 x 3		3 : 1 *or* 3			$\frac{\text{New Surface Area}}{\text{Original Surface Area}} =$ ⇒
B	8 x 6 x 10		1 : 5 *or* $\frac{1}{5}$			$\frac{\text{New Surface Area}}{\text{Original Surface Area}} =$ ⇒
C	5 x 6 x 4		3 : 2 *or* $\frac{3}{2}$			$\frac{\text{New Surface Area}}{\text{Original Surface Area}} =$ ⇒
D	4 x 18 x 12		2 : 3 *or* $\frac{2}{3}$			$\frac{\text{New Surface Area}}{\text{Original Surface Area}} =$ ⇒

Complete: surface area scale factor = (linear scale factor)

Complete the following volume scale factor for the following rectangular prisms.

	Original Dimensions (cm)	Original Volume (cm^3)	Linear Scale Factor	New Dimensions (cm)	New Volume (cm^3)	Volume Scale Factor New Volume : Original Volume $\frac{\text{New Volume}}{\text{Original Volume}}$
A	4 x 6 x 3		3 : 1 *or* 3			$\frac{\text{New Volume}}{\text{Original Volume}} =$ ⇒
B	8 x 6 x 10		1 : 5 *or* $\frac{1}{5}$			$\frac{\text{New Volume}}{\text{Original Volume}} =$ ⇒
C	5 x 6 x 4		3 : 2 *or* $\frac{3}{2}$			$\frac{\text{New Volume}}{\text{Original Volume}} =$ ⇒
D	4 x 18 x 12		2 : 3 *or* $\frac{2}{3}$			$\frac{\text{New Volume}}{\text{Original Volume}} =$ ⇒

Complete: volume scale factor = (linear scale factor)

Math 2201 Proportional Reasoning Unit 8 Lesson 5

Scale Factor Summary

Linear scale factor is a ratio in the form $a : b$ or $\frac{a}{b}$

Calculated using $\frac{\text{New Length}}{\text{Original Length}}$

Perimeter scale factor is a ratio in the form $a : b$ or $\frac{a}{b}$

Calculated using $\frac{\text{New Perimeter}}{\text{Original Perimeter}}$

Area scale factor is a ratio in the form $a : b$ or $\frac{a}{b}$

Calculated using $\frac{\text{New Area}}{\text{Original Area}}$

Surface Area scale factor is a ratio in the form $a : b$ or $\frac{a}{b}$

Calculated using $\frac{\text{New Surface Area}}{\text{Original Surface Area}}$

Volume scale factor is a ratio in the form a : b or $\frac{a}{b}$

Calculated using $\frac{\text{New Volume}}{\text{Original Volume}}$

The Scale Factor Relationships

Perimeter	Perimeter Scale Factor = Linear Scale Factor
Area	Area Scale Factor = (Linear Scale Factor)^2
Surface Area	Surface Area Scale Factor = (Linear Scale Factor)^2
Volume	Volume Scale Factor = (Linear Scale Factor)^3

Math 2201 Proportional Reasoning Unit 8 Lesson 5

Magic's Basketball Camp uses basketballs with a diameter of 12.8 cm. The camp organiser, uses an advertisement basketball with diameter of 2.5 m to put on the roof of their building.

a) The surface area and volume calculations of the basketball with a diameter of 12.8 cm are shown below. Complete the work below to calculate the surface area, to the nearest 0.1 cm^2, and the volume, to the nearest 0.1 cm^3, of the basketball with a diameter of 2.5 m.

b) Use the above results to determine, to the nearest whole number, the following scale factors from the 12.8 cm basketball to the 2.5 m basketball.

linear scale factor surface area scale factor volume scale factor

c) Show how to use the volume scale factor to determine the linear scale factor.

d) Show how to use the volume scale factor to determine the area scale factor.

If a question asks for the **scale factor** of an enlargement or reduction, it is implied that the **linear scale factor** is required.

Important: when the dimensions of similar 3-D objects are related by a scale factor *k*, their surface areas are related by k^2 and their volumes are related by k^3