Properties of Graphs of Quadratic Functions

Goal: **Identify the characteristics of graphs of quadratic functions:**

- **Vertex**
- **Intercepts**
- **Domain and Range**
- **Axis of Symmetry**

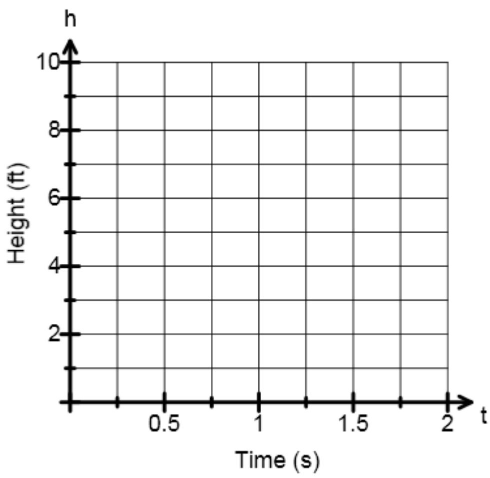
and use the graphs to solve problems.

(I) Using symmetry to estimate the coordinates of a point

Example: The height of a volleyball against time

Nicolina plays on her school's volleyball team. At a recent match, her Nonno, Marko, took some time-lapse photographs while she warmed up. He set his camera to take pictures every 0.25 s. He started his camera at the moment the ball left her arms during a bump and stopped the camera at the moment that the ball hit the floor.

(i) At what time did the volleyball reach its greatest height?



Time (s)	Height (ft)
0.00	2
0.25	6
0.50	8
0.75	8
1.00	6
1.25	2

(ii) **Use the graph to approximate the greatest height?**

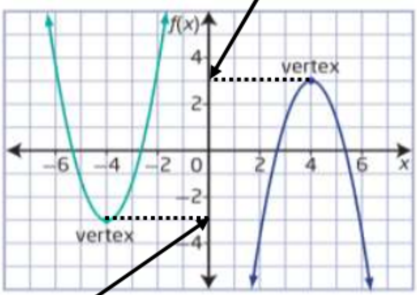
(iii) **Using the answers to (i) and (ii), what are the coordinates of the highest point?**

Vertex:

The point at which the quadratic function reaches its maximum or minimum value.

Maximum Value

- When the graph opens **down**, the vertex is the ________ point on the graph and the *y*-coordinate of the vertex is the ____________ value.



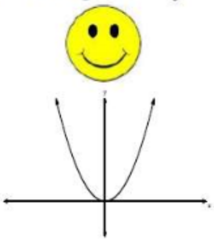
Minimum Value

- When the graph opens **up** the vertex is the ________ point on the graph and the y-coordinate is the _____________ value.

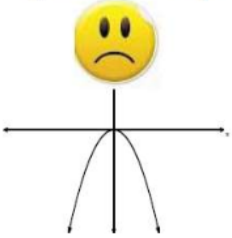
The value of 'a' in a quadratic function

$y = ax^2 + bx + c$ determines whether the vertex is a maximum or minimum point.

Positive Quadratic ($y = x^2$)



Negative Quadratic ($y = -x^2$)



(II) Attaining a maximum or minimum value through a table of values or graph

Example:
A golf ball is struck and its height with respect to time is represented by the function $h(t) = -3t^2 + 12t$ where $h(t)$ represents height and t is the time in seconds.

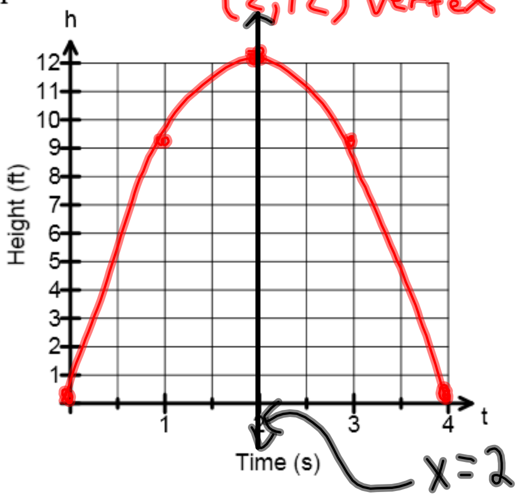
(a) What's the direction of opening? DOWN

(b) Will the ball attain a $\boxed{\text{maximum}}$ or ~~minimum~~ height?

(c) What's the y – intercept? $(0,0)$ $\boxed{t \geq 0}$

(d) Create a table of values and graph the function. $(2,12)$ vertex

t	h(t)
0	0
1	9
2	12
3	9
4	0



(e) How long does it take for the ball to attain its maximum height?

$t = 2$ sec

(f) What is the maximum height attained?

12 ft

(g) What are the coordinates of the vertex?

$(2,12)$

(III) Attaining a maximum or minimum value through a graph and formula

For each function indicated in the table determine:

(i) the vertex (using graphing software https://www.desmos.com/)
(ii) the equation of axis of symmetry
(iii) the values of 'a', 'b' and 'c' from the function $y = ax^2 + bx + c$
(iv) the value of $-\frac{b}{2a}$

Function $y = ax^2 + bx + c$	**Vertex**	**Equation of Axis of Symmetry**	**a**	**b**	**c**	$-\frac{b}{2a}$
$y = x^2 - 4x + 7$	(2, 3)	$x = 2$	1	-4	7	2
$y = -2x^2 - 4x + 7$	(-1, 9)	$x = -1$	-2	-4	7	-1
$y = 3x^2 - 6x + 10$	(1, 7)	$x = 1$	3	-6	10	1

$\frac{-b}{2a}$

$\frac{-(-6)}{2(3)}$

$\frac{6}{6}$

(a) What do you notice about the x– coordinate of each vertex, the equation of axis of symmetry and the value of $-\frac{b}{2a}$?

Same value for each function

(b) Once the x– coordinate of the vertex is attained from a quadratic function such as $y = -2x^2 - 4x + 7$, how could we algebraically attain the y– coordinate?

plugging the x back into the function.

Summary Attaining the vertex of a quadratic function $y = ax^2 + bx + c$

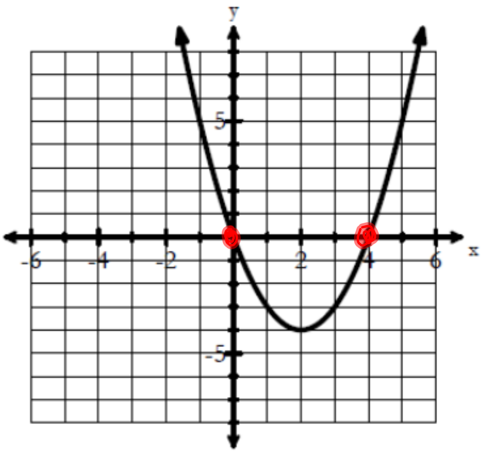
(i) Get the x– coordinate of the vertex by the formula $x = -\frac{b}{2a}$

(ii) Substitute that result into $y = ax^2 + bx + c$ to attain the y– coordinate.

The vertex and equation of axis of symmetry of a quadratic function can be attained:

(A) Graphically

Example: State the vertex and equation of axis of symmetry.



Vertex: (2,-4)

Equation of axis of symmetry: x=2

(B) Tabulation

Example: State the vertex and equation of axis of symmetry.

x	-1	0	1	2	3
y	10	1	-2	1	10

Vertex: (1,-2)

Equation of axis of symmetry: x=1

(C) Algebraically

Example: Determine the vertex and equation of axis of symmetry for $y = 2x^2 - 8x + 7$.

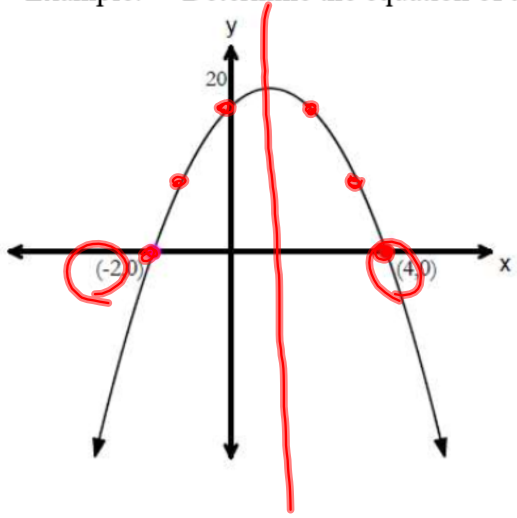
$$x = -\frac{b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2,$$ so $x=2$ is A. of S.

$$y = 2(2)^2 - 8(2) + 7$$
$$y = 8 - 16 + 7 = -1$$

vertex $(2,-1)$

) Determining the axis of symmetry from a set of points.

Example: Determine the equation of axis of symmetry from the parabola.



$$\frac{-2+4}{2} = \frac{2}{2} = 1$$

Where is the line of symmetry positioned compared to the location of the two given points?

In the middle

Summary	Axis of symmetry
	(i) A vertical reflection line that passes through the vertex
	(ii) Can be attained by the formula $x = -\frac{b}{2a}$ when the quadratic function $y = ax^2 + bx + c$ is given.
	(iii) Can be attained from two points with the same y–coordinate by averaging the x–coordinates.

Example: Determine the equation of axis of symmetry for each parabola that contains the points:

(a) (–2, 4) and (6, 4)

$\frac{-2+6}{2} = \frac{4}{2} = 2$

A.of S. $\boxed{x = 2}$

(b) (5, 0) and (11, 0)

$x = 8$

(V) Attaining the domain and range of a quadratic function

Review of Domain and Range

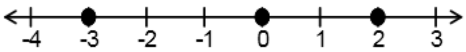
Domain
- is the set of all input values (or x–values)

Range
- is the set of all output values (or y–values)

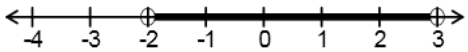
The domain and range can be attained:
(i) Graphically
(ii) Tabulation (or set of points)
(iii) Function

Determining Domain & Range Graphically

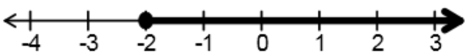
1. State the domain using set notation for:

(a) 

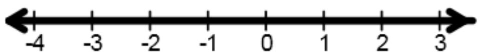
Set notation: $\{x \mid x = -3, 0, 2, x \in I\}$

(b) 

Set notation: $\{x \mid -2 < x < 3, x \in \mathbb{R}\}$

(c) 

Set notation: $\{x \mid x \geq -2, x \in \mathbb{R}\}$

(d) 

Set notation: $\{x \mid x \in \mathbb{R}\}$

2. State the range using set notation for:

(a) 

Set notation: $\{y \mid y \geq -3$

(b) 

Set notation: $\{y \mid y \leq -1$

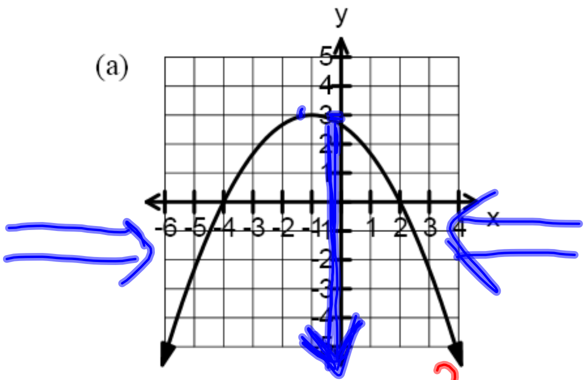
3. State the domain and range for:

NOTE:

For Domain - all points on the graph FALL to the x-axis

For Range - all points on the graph MOVE OVER to the y-axis

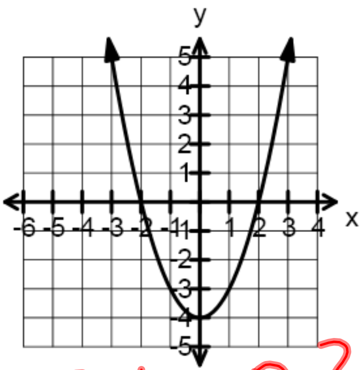
(a)



Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \le 3, y \in \mathbb{R}\}$

(b)



Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \ge -4, y \in \mathbb{R}\}$

Determining Domain & Range from a Quadratic Function

How do we attain the domain of a quadratic function such as $y = -2x^2 + 4x + 1$ without the aid of a graph?

(a) What is the direction of opening for the given function?

(b) Will the function have a maximum or minimum value?

(c) How can we algebraically attain the maximum/minimum value?

(d) How does the above information enable us to express the range?

SUMMARY: **To attain the domain and range from $y = ax^2 + bx + c$**

Domain – for any quadratic function is ______________

Range (i) determine the ______________ of opening

(ii) determine the _______________of vertex by ________

(iii) Substitute the result from (ii) into the function

_________________to get the maximum/minimum value

(iv) State the range.

Example: Determine the domain and range for:

(a) $y = 3x^2 - 2$ (b) $y = x^2 + 4x + 4$

(c) $y = -x^2 + 6x - 8$

P.332 – 335 #2 (state the y–intercept), #4, #5, #6, #7a(i), b, c, #9 b, c, #10, #11b, c
#13a, b #14