6.5 Maximum/Minimum Word Problems

To solve Max/Min Problems you will have to determine the highest (or lowest) point , in other words, **the vertex**.

(A) Determining the maximum height given the quadratic function.

Example 1:

A boat in distress fires off a flare. The height of the flare, *h*, in metres above the water, *t* seconds after shooting, is modeled by the function

$h(t) = -4.9t^2 + 29.4t + 3$.

Algebraically determine the maximum height attained by the flare.

Solution: get x-coordinate of vertex using $-\frac{b}{2a}$ $\left(t = -\frac{b}{2a}\right)$

$$t = \frac{-(29.4)}{2(-4.9)} = \frac{-29.4}{-9.8} = 3$$

(how long)

use $t = 3$ to get max. height

$$h(3) = -4.9(3)^2 + 29.4(3) + 3$$
$$= -4.9(9) + 88.2 + 3$$
$$= -44.1 + 88.2 + 3$$
$$= 47.1$$

So, max height of flare is 47.1 m

Example 2:

The path of a volleyball is given by $h(t) = -\frac{1}{2}t^2 + 5t + 2$ where t is the time in seconds and h is the height in meters. At what time does the ball reach its maximum height?

$$t = \frac{-b}{2a} = \frac{-(5)}{2(-\frac{1}{2})} = \frac{-5}{-1} = 5$$

So, it reaches its max height at $t = 5$ sec

Please note we do not have to go further to substitute it back into the equation to find the height. This question asks just when it happens.

Be very careful to INTREPRET the question correctly!

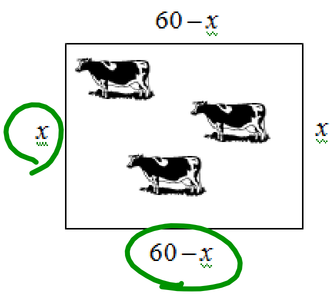
(B) Area Problems

(i) Open Field

Example 1:

A farmer is constructing a rectangular fence in an open field to contain cows. There is 120 m of fencing.

(a) Write the quadratic function that models the region. Area = length × width



$$A = (60 - x)x \quad \text{OR} \quad A = x(60 - x)$$

$$A = 60x - x^2$$

$$\boxed{A = -x^2 + 60x}$$

vertex

(b) Determine the maximum area of the enclosed region.

$A = -x^2 + 60x$

use $x = \frac{-b}{2a} = \frac{-(60)}{2(-1)} = \frac{-60}{-2} = 30$

now use $x = 30$ in $A = -x^2 + 60x$ to get max area

So $A = -(30)^2 + 60(30)$

$A = -900 + 1800$

$A = 900$

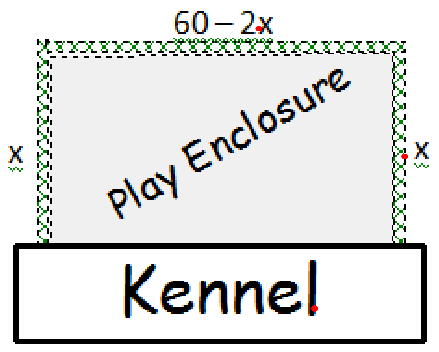
So max area is $900 m^2$

(i) Using a Physical Structure as One Side

Example 1:

A rectangular play enclosure for some dogs is to be made with 60 m of fencing using the kennel as one side of the enclosure as shown.

(a) Algebraically determine the quadratic function that models the area.



$A = (60 - 2x)x$

OR

$A = x(60 - 2x)$

$A = 60x - 2x^2$

$A = -2x^2 + 60x$

vertex

(b) Use the function to determine the maximum area.

$$A = -2x^2 + 60x$$

$$x = \frac{-b}{2a} = \frac{-60}{2(-2)} = \frac{-60}{-4} = 15$$

$$A = -2(15)^2 + 60(15)$$

$$A = -2(225) + 900$$

$$A = -450 + 900$$

$$A = 450$$

Max Area is $450\,m^2$

(c) State the domain and range of the variables in the function.

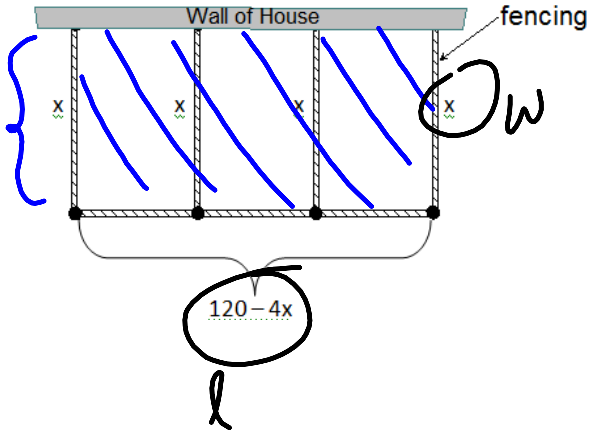
$$D: \{x \mid 0 < x < 30, x \in \mathbb{R}\}$$

$$R: \{A \mid 0 < A \leq 450, A \in \mathbb{R}\}$$

Example 2:

A rectangular region, placed against the wall of a house, is divided into three regions of equal area using a total of 120 m of fencing as shown.

(a) Algebraically develop a quadratic function that models the area.



$A = x(120-4x)$

$A = 120x - 4x^2$

$A = -4x^2 + 120x$

(b) Determine the maximum area of the pen. $A = -4x^2 + 120x$

$$x = -\frac{b}{2a} = \frac{-(120)}{2(-4)} = \frac{-120}{-8} = 15$$

$$A = -4(15)^2 + 120(15)$$

$$A = -4(225) + 1800$$

$$A = 900\,m^2$$

(c) State the domain and range of the variables in the function.

Domain: $\{x \mid 0 < x < 30, x \in \mathbb{R}\}$

Range: $\{A \mid 0 < A \leq 900, y \in \mathbb{R}\}$

(C) Revenue Problems

Formula for Revenue:

Revenue = (Number Sold)(Cost)

Example 1:

Global Gym charges its adult members \$50 monthly for a membership. The club has 600 adult members. Global Gym estimates that for each \$5 increment in the monthly fee, it will lose 50 members.

(a) Determine the function that models Global Gym's revenue.

$$R = (600 - 50x)(50 + 5x)$$

$$R = 30000 + 3000x - 2500x - 250x^2$$

$$\boxed{R = -250x^2 + 500x + 30000}$$

(b) Determine the maximum revenue generated.

$$R = -250x^2 + 500x + 30000$$

$$x = -\frac{b}{2a} = \frac{-500}{2(-250)} = \frac{-500}{-500} = 1$$

$$R = -250(1)^2 + 500(1) + 30000 = 30250$$

(c) Determine the monthly fee that will produce the greatest revenue.

$$fee = 50 + 5x$$

$$fee = 50 + 5(1)$$

$$\boxed{fee = 55}$$

Example 2:

An orange grower has 400 crates of oranges ready for market and will have 20 more crates each day that shipment is delayed. The present price is $60 per crate however, for each day shipment is delayed, the price per crate decreases by $2.

(a) Determine the revenue function that models this function.

$$R = \overbrace{(400 + 20x)}^{\text{number}}\overbrace{(60 - 2x)}^{\text{price}}$$

$$R = 24\,000 - 800x + 1200x - 40x^2$$

$$R = -40x^2 + 400x + 24\,000$$

$$R = -40x^2 + 400x + 24000$$

(b) Determine the maximum revenue that can be generated.

$$x = \frac{-b}{2a} = \frac{-(400)}{2(-40)} = \frac{-400}{-80} = 5$$

$$R = -40(5)^2 + 400(5) + 24000 = 25\,000$$

(c) Determine the price per crate that will produce the greatest revenue.

$$\begin{aligned} \text{price} &= 60 - 2x \\ &= 60 - 2(5) \\ &= 50 \end{aligned}$$

D) Number Connections

***Example 1*:**

Two numbers have a difference of 8. What are the two numbers if their product is a minimum?

Solution: Let x be the larger number then $x-8$ is smaller number.

$$P = x(x-8)$$

$$P = x^2 - 8x$$

use $x = -\frac{b}{2a} = -\frac{(-8)}{2(1)} = \frac{8}{2} = 4$

other number is $x-8$

$= 4-8$

$= -4$

***Example 2*:**

The sum of two numbers is 60. Algebraically determine the two numbers if their product is a maximum.

Solution: Let x be one number
then $60 - x$ is other number

$$P = x(60 - x)$$
$$P = 60x - x^2$$
$$P = -x^2 + 60x$$

use $x = -\frac{b}{2a} = -\frac{(60)}{2(-1)} = \frac{-60}{-2} = 30$

other number is $60 - x = 60 - 30 = 30$

***Example 3*:**

The sum of two numbers is 28. Algebraically determine the two numbers if the sum of their squares is a minimum.

Solution: Let x be one number
then $28-x$ is other number

$$S = (x)^2 + (28-x)^2$$

$$S = x^2 + (28-x)(28-x)$$

$$S = x^2 + 784 - 28x - 28x + x^2$$

$$S = 2x^2 - 56x + 784$$

use: $x = -\frac{b}{2a} = -\frac{(-56)}{2(2)} = \frac{56}{4} = 14$

other number is $28 - x$
$= 28 - 14$
$= 14$

Two numbers are 14 and 14