UNIT V: Applications of Derivatives

3.1 Maximum and minimum values

Graphing: intercepts and Extrema

Extrema → max/min points

Curves can have absolute max/min points

→ no other point, anywhere on the curve, higher (abs. max) or lower (abs. min)

Curves can have relative (local) max/min points.

→ rel. max is highest compared to points which are nearby

→ rel. min is lowest compared to points which are nearby.

Extrema occur at what are called critical points (values) ⇒ $f'(x)=0$ OR $f'(x)$ is undefined

All Extrema are critical points, but not all critical points are extrema

To get critical points, determine the derivative, set it equal to zero and solve for x. (or NPVs)

Ex: Determine extrema of $f(x) = x^3 - 3x^2$

1st get critical points (values)
x only

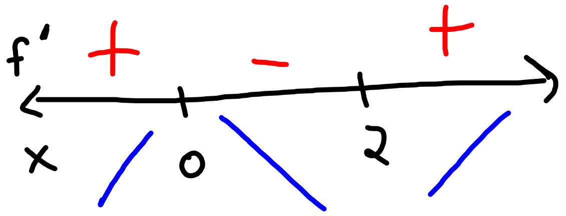
$f'(x) = 3x^2 - 6x$

set $f'(x) = 0$ → $3x^2 - 6x = 0$
solve for x $3x(x-2) = 0$
$x = 0$ OR $x = 2$

To determine if max or min test slope on either side of critical points.

Could use number line.

Slope →



$f'(x) = 3x^2 - 6x$

since f' changes from pos to neg at $x=0$ it is a rel max (0,0)

Since f' changes from neg to pos at $x=2$ it is a rel min (2, −4)

Ex: sketch $f(x) = x^4 - 6x^2 + 8$ identifying all intercepts and extrema

Intercepts: y-intercept $(0, 8)$

x-intercepts: set $f(x)=0$ and solve

$$x^4 - 6x^2 + 8 = 0$$
$$(x^2-4)(x^2-2) = 0$$
$$x^2-4=0 \text{ OR } x^2-2=0$$
$$x = \pm 2 \qquad x = \pm\sqrt{2}$$

Critical #s : set $f'(x)=0$ and solve

$$f(x) = x^4 - 6x^2 + 8$$
$$f'(x) = 4x^3 - 12x$$

solve $4x^3 - 12x = 0$ $\qquad x = 0,\ x = -\sqrt{3},\ x = \sqrt{3}$

$$x^3 - 3x = 0$$
$$x(x^2-3) = 0$$
$$x = 0 \qquad x^2 - 3 = 0$$
$$x = \pm\sqrt{3}$$

$$f'(x) = 4x^3 - 12x$$

Test slopes: f' — + — +

x: $-\sqrt{3}$, 0, $\sqrt{3}$

So rel min at $x = -\sqrt{3}$ and $x = \sqrt{3}$
$(-\sqrt{3}, -1)$ $(\sqrt{3}, -1)$

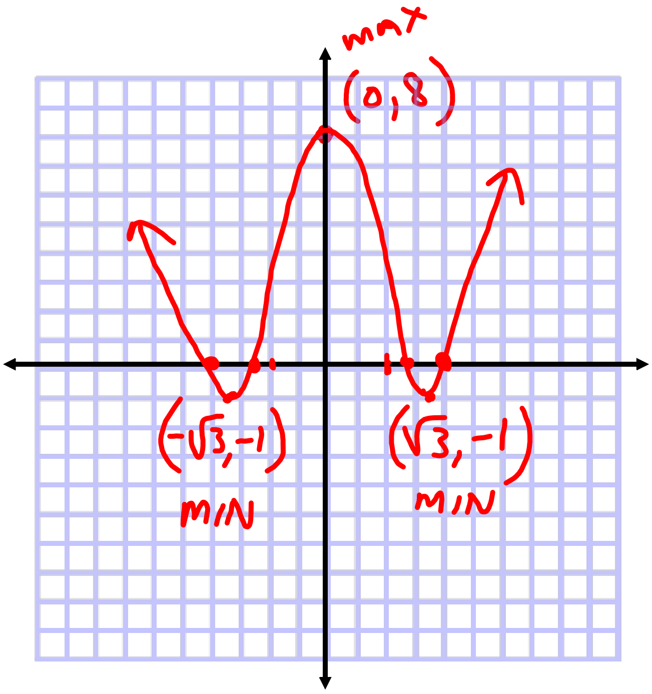
rel max at $x = 0 \rightarrow (0, 8)$

When $x = -\sqrt{3}$ $f(-\sqrt{3}) = (-\sqrt{3})^4 - 6(-\sqrt{3})^2 + 8$

$= 9 - 18 + 8$

$= -1$

When $x = \sqrt{3}$ $f(\sqrt{3}) = -1$



Ex: Sketch $f(x) = 3x^4 - 4x^3 - 12x^2$

Student Example

Sketch $f(x)=3x^4-4x^3-12x^2$

y-int: $(0,0)$

x-int: let $f(x)=0$

$$3x^4-4x^3-12x^2=0$$

$$x^2(3x^2-4x-12)=0$$

$\boxed{x=0 \text{ (double root)}}$ $\quad x=\dfrac{4\pm\sqrt{(-4)^2-4(3)(-12)}}{2(3)}$

$$x=\frac{4\pm\sqrt{160}}{6}$$

$$x=\frac{4\pm4\sqrt{10}}{6}=\boxed{\frac{2\pm2\sqrt{10}}{3}\approx\begin{matrix}2.8\\-1.4\end{matrix}}$$

Critical Points:

$f(x)=3x^4-4x^3-12x^2$

$f'(x)=12x^3-12x^2-24x$

$0=12x^3-12x^2-24x$

$0=12x(x^2-x-2)$

$0=(12x)(x-2)(x+1)$

CP at $x=-1,0,2$

f'	$-$	$+$	$-$	$+$
x	-1	0	2	
	↘	↗	↘	↗

$\therefore x=-1$ is local min

$x=0$ is local max

$x=2$ is local min

At $x=0$, $f(0)=0$

At $x=-1$, $f(-1)=-5 \Rightarrow (-1,-5)$

At $x=2$, $f(2)=-32 \Rightarrow (2,-32)$