

6.3 Factored form of a quadratic

Now look at quadratic function in

the form $y = a(x-r)(x-s)$ factored form

Ex: $y = 2(x-3)(x+1)$

to get x -intercepts make $y=0$

$$\frac{0}{2} = \frac{2(x-3)(x+1)}{2}$$

$$0 = (x-3)(x+1)$$

so either $x-3=0$ OR $x+1=0$
 $x=3$ OR $x=-1$

Zero product property

If $a \cdot b = 0$

either $a=0$ or $b=0$

or $a=b=0$

So x -intercepts are $(3,0)$ and $(-1,0)$

and axis of symmetry is $x = \frac{3+(-1)}{2} = \frac{2}{2} = 1$

thus vertex is $(1, -8)$

$$y = 2(1-3)(1+1)$$

$$y = 2(-2)(2) = -8$$

*middle of
x-intercepts*

$$\underline{\text{Ex:}} \quad y = (x+2)(x-3)$$

x-intercepts \Rightarrow make $y=0$

$$0 = (x+2)(x-3)$$

$$x+2=0 \quad x-3=0$$

$$x=-2 \quad x=3$$

called "zeros"
The values of x
that make $y=0$

Axis of symmetry: $x = \frac{-2+3}{2} = \frac{1}{2}$

$$\Rightarrow x = \frac{1}{2}$$

get $y \Rightarrow$

$$y = \left(\frac{1}{2}+2\right)\left(\frac{1}{2}-3\right)$$

$$y = \left(\frac{1}{2}+\frac{4}{2}\right)\left(\frac{1}{2}-\frac{6}{2}\right)$$

$$y = \left(\frac{5}{2}\right)\left(-\frac{5}{2}\right)$$

$$y = -\frac{25}{4} \quad \text{so vertex is } \left(\frac{1}{2}, -\frac{25}{4}\right)$$

y-coordinate
of vertex

y-intercept \Rightarrow make $x=0$

$$y = (0+2)(0-3) = (2)(-3) = -6$$

$$\text{Domain: } \{x \mid x \in \mathbb{R}\}$$

$$\text{Range: } \{y \mid y \geq -\frac{25}{4}, y \in \mathbb{R}\}$$

opens up

Ex: Determine all characteristics of

$$y = -3(x-1)(x+5)$$

y-intercept: make $x=0$

$$y = -3(0-1)(0+5)$$

$$y = -3(-1)(5)$$

$$y = 15 \Rightarrow (0, 15)$$

x-intercepts: make $y=0$

$$0 = \frac{-3}{-3} \frac{-3(x-1)(x+5)}{-3}$$

$$0 = (x-1)(x+5)$$

$$(x-1)=0 \quad (x+5)=0$$

$$x=1 \quad x=-5$$

$$(1, 0) \quad (-5, 0)$$

$$D: \{x \mid x \in \mathbb{R}\}$$

$$R: \{y \mid y \leq 27, y \in \mathbb{R}\}$$

opens down
vertex is MAX

axis of symm
vertex
y-intercept
x-intercepts
Domain
Range
direction of opening
max or min?

Axis of Symmetry

$$x = \frac{-5+1}{2} = \frac{-4}{2}$$

$$x = -2$$

y-coordinate of vertex

$$y = -3(-2-1)(-2+5)$$

$$y = -3(-3)(3)$$

$$y = 27$$

vertex $(-2, 27)$

p. 346 #s 1-5

Ex: Determine the characteristics of
 $y = -3x^2 + 6x + 9$ and sketch
 the graph.

Solution: y-intercept (0,9)

x-intercepts \Rightarrow make $y=0$

$$0 = \frac{-3x^2 + 6x + 9}{-3}$$

$$0 = x^2 - 2x - 3$$

$$0 = (x+1)(x-3)$$

$$x+1=0 \text{ OR } x-3=0$$

$$x=-1 \quad x=3$$

$$\begin{array}{r} -3 \\ -1 \overline{) 3} \\ 1 \quad -3 \end{array}$$

Note: factored form of function is:

$$y = -3(x+1)(x-3)$$

Axis of symmetry: Average the x-intercepts

$$x=1$$

$$x = \frac{-1+3}{2} = \frac{2}{2} = 1$$

vertex is (1,12) \Rightarrow get y by subbing $x=1$ into function.

$$D: \{x | x \in \mathbb{R}\}$$

$$R: \{y | y \leq 12, y \in \mathbb{R}\}$$

↑
opens down
vertex is max!

$$y = -3(x+1)(x-3)$$

$$y = -3(1+1)(1-3)$$

$$y = -3(2)(-2)$$

$$y = 12$$