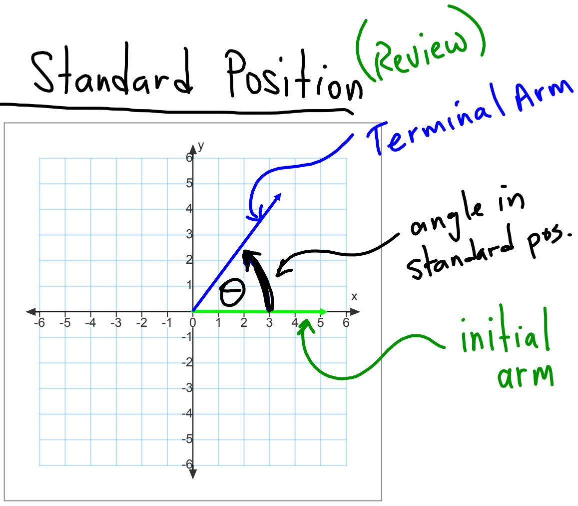
UNIT 4: Trigonometry and The UNIT CIRCLE

4.1 Angles and angle measure

Angles in Standard Position (Review)



Positive angles → rotate counterclockwise
Negative angles → rotate clockwise

Reference angle → angle formed between terminal arm and nearest x-axis. Always between 0° and 90°

Co-terminal Angles → Angles which share same terminal arm

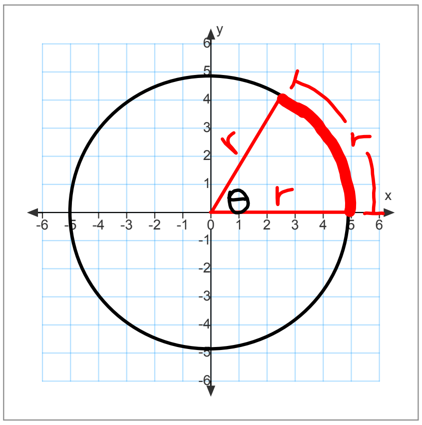
Quadrantal angles → terminal arm lies on either the x or y axis.

Co-Terminal angles → Angles which share the same terminal arm. Always differ by multiples of 360°

Principle Angle → Smallest Positive co-terminal angle

Radian Measure

when the terminal arm rotates to a point where the length of the arc (piece of the circle) is exactly the same length as the radius, then that angle is exactly one radian.



This will happen approximately 6.28 times in one full rotation of a circle.

To be exact it happens 2π times

i.e. one full rotation of a circle is 2π radians.

Now we can say that

$360^\circ = 2\pi$ radians (one full rotation)

$180^\circ = \pi$ ← nothing written implies radians

$90^\circ = \frac{\pi}{2}$

$45^\circ = \frac{\pi}{4}$

$60^\circ = \frac{\pi}{3}$

$30^\circ = \frac{\pi}{6}$

VERY POPULAR

To convert from degrees to radians we can use the fact that $180° = \pi$ and multiply by $\frac{\pi}{180°}$ (like × by 1)

Ex: $90° = 90° \times \frac{\pi}{180°} = \frac{90\pi}{180} = \frac{\pi}{2}$

$20° = 20° \times \frac{\pi}{180°} = \frac{20\pi}{180} = \frac{\pi}{9}$

$45° = 45° \times \frac{\pi}{180} = \frac{45\pi}{180} = \frac{\pi}{4}$

To convert from radians to degrees flip to $\frac{180}{\pi}$

Ex: $\frac{\pi}{6} = \frac{\cancel{\pi}}{6} \times \frac{180°}{\cancel{\pi}} = \frac{180°}{6} = 30°$

Ex: $\frac{3\pi}{4} = \frac{3\cancel{\pi}}{\cancel{4}_{1}} \times \frac{\cancel{180}^{45}}{\cancel{\pi}} = 135°$

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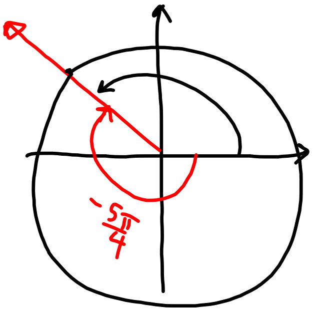
Sketch $-\frac{5\pi}{4}$



Are $120°$ and $-240°$ coterminal?

Yes, since $-240° + 360° = 120°$

Tell me one angle that is coterminal with $60°$

Ans: $-300°$ $\{60° - 360° = -300°$

Ans: $420°$ $\{60 + 360 = 420°$

Ans: $780°$ $\{60 + 360(2) = 780°$

Now write all angles that are coterminal with $60°$

$\{60° + 360k, k \in I\}$ $\{\frac{\pi}{3} + 2\pi k, k \in I\}$

coterminal with $90°$ between -720 and 720

Such that $-720 \le \theta \le 720$

$90° + 360 = 450$

$450 + 360 = \cancel{810}$

$90° - 360° = -270°$

$-270° - 360° = -630°$

$\{450°, -270°, -630°$

Arc Length

Arc length (s)

$s = \theta r$ where

θ is angle in radians

r is the radius

when you cut a circle with an angle θ, the ratio of the arc length to the circumference is equal to the ratio of θ to the entire circle of 2π

so $\frac{\text{arc length}}{\text{circumference}} = \frac{\theta}{2\pi}$

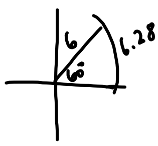
$$\cancel{2\pi r} \cdot \frac{s}{\cancel{2\pi r}} = \frac{\theta}{\cancel{2\pi}} \cdot \cancel{2\pi} r$$

$$\boxed{s = \theta \cdot r}$$

Ex: what is the arc length formed by an angle of $\frac{\pi}{3}$ in a circle with a diameter of 12 cm?

Solution: $s = \theta r$

$s = \frac{\pi}{3} \cdot 6 = 2\pi \doteq 6.28\text{cm}$



Ex: what angle, in radians, would generate an arc length of 15.3cm in a circle with radius 4cm?

Solution: $s = \theta r$

$\frac{15.3}{4} = \frac{\theta(4)}{4}$

$3.83 = \theta$

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