Section 6.5: Financial Applications Involving Exponential Functions

1. Simple Interest:

Simple interest is calculated only in terms of the original amount invested, not on the accumulated interest.

Simple Interest Formula: $\boxed{A = P + Prt}$ or $\boxed{A = P(1 + rt)}$

where P is the principal amount

t is the time in years

r is the interest rate per annum (as a decimal)

NOTE: *A* is the sum of the principal (*P*) and the accumulated interest (*Prt*)

Example 1:

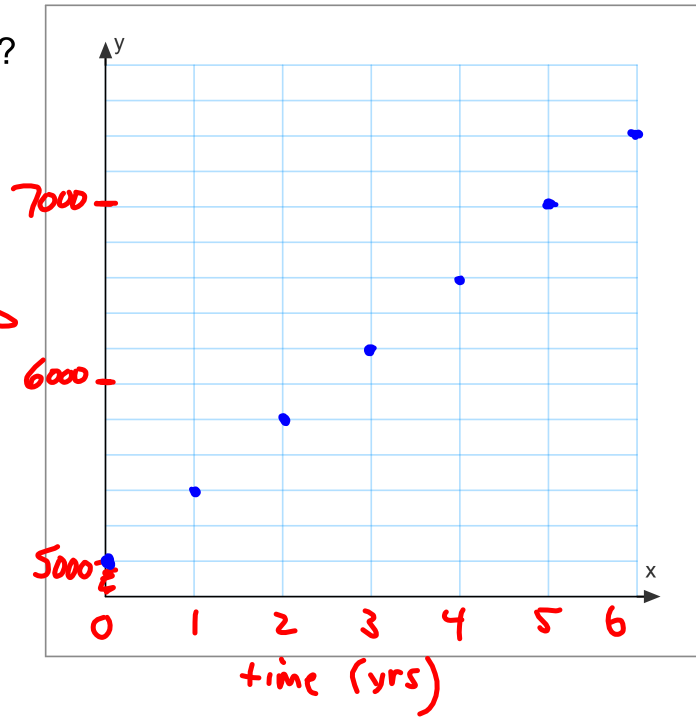
Kyle invested his summer earnings of $5000.00 at 8% simple interest, paid annually.

a) Create a table of values and graph the growth of the investment for 6 years using time, in years, as the domain and the value of the investment as the range.

Time (years)	Value of Investment ($)
0	5000
1	$5000 + 5000(0.08)(1) = 5400$
2	$5000 + 5000(0.08)(2) = 5800$
3	$5000 + 5000(0.08)(3) = 6200$
4	6600
5	7000
6	7400

a) What does the shape of the graph tell you about the type of growth?

linear graph, so growth is constant
goes up by same amount $ every year.



b) Why is the data discrete?

Annual interest rate paid once per year.

c) What do the y-intercept and slope represent for the investment?

y-intercept ⇒ initial money invested
slope ⇒ rate at which money is growing

d) What is the value of the investment after 10 years?

$A = P + Prt$

$A = 5000 + 5000(0.08)(10)$

$A = 9000$

e) How much interest was earned after 10 years?

\$4000

2. Compound Interest:

Compound interest is determined by applying the interest rate to the sum of the principal and any accumulated interest.

Compound Interest Formula: $\boxed{A = P(1+i)^n}$

where *A* is the future value

P is the principal amount

i is the interest rate **per compounding period** (expressed as a decimal)

t is the time in years

n is the **number of compounding periods**

n is NOT the number of years!

Refer to previous example of $5000.00 (*P*) in a savings account earning an annual interest of 8%.

Time (years)	Amount of Annual Interest	Value of Investment ($)
0	0	5000
1	400	$5000(1.08)^1 = 5400$
2	432	$5000(1.08)^2 = 5832$
3	466.56	$5000(1.08)^3 = 6298.56$

NOTE: The accumulated interest and the value of the investment do not grow by a constant amount as they do with simple interest.

An exponential regression to model the investment would result in the equation: $y = 5000(1.08)^x$

Note how this compares to: $A = P(1+i)^n$.

Investments can also have daily, weekly, monthly, quarterly, semi-annually, or annually compounding periods.

Compounding Period	Number of Times Interest is Paid	Interest Rate per Compounding Period i
Daily	365 times per year	$i = \frac{\text{annual rate}}{365}$
Weekly	52 times per year	$i = \frac{\text{annual rate}}{52}$
Bi-Weekly	26 times per year	$i = \frac{\text{annual rate}}{26}$
Semi-monthly	24 times per year	$i = \frac{\text{annual rate}}{24}$
Monthly	12 times per year	$i = \frac{\text{annual rate}}{12}$
Quarterly	4 times per year	$i = \frac{\text{annual rate}}{4}$
Semi-annually	2 times per year	$i = \frac{\text{annual rate}}{2}$
Annually	1 time per year	$i = \frac{\text{annual rate}}{1}$

$A = P(1+i)^n$

For example, if \$5000 is invested at 6% compounded monthly,

12

$$i = \frac{\text{annual rate}}{12} = \frac{0.06}{12} = 0.005$$

The compound interest formula is defined as:

$$A = 5000(1.005)^n$$

where n is the number of months, NOT the number of years

Example 2: Complete the table if the interest rate is 4.8% per year.

0.048

Compounding Period	Number of Times Interest is Paid	Interest Rate per Compounding Period (i)
Bi-Monthly	6	$i = \frac{0.048}{6} = 0.008$
Monthly	12	$i = \frac{0.048}{12} = 0.004$
Quarterly	4	$i = \frac{0.048}{4} = 0.012$
Semi-Annually	2	$i = \frac{0.048}{2} = 0.024$
Annually	1	$i = \frac{0.048}{1} = 0.048$

Example 3:

If \$5000 is invested, calculate A (the future value) using $A = P(1+i)^n$ for each situation.

a) 11% per year, compounded quarterly for 3 years

$\hookrightarrow 0.11$ $\quad 4$ $\quad n = 12$

$i = \frac{0.11}{4} = 0.0275$

$A = 5000(1.0275)^{12} = \$6923.92$

b) 6.5% per year, compounded semi-annually for 3 years

2

$A = 5000\left(1 + \frac{0.065}{2}\right)^{2\times3}$

$A = 5000(1.0325)^{6} = \$6057.74$

c) 15.6% per year, compounded monthly for 2 years

$A = 5000\left(1 + \frac{0.156}{12}\right)^{12\times2}$

$A = 5000(1.013)^{24} = \$6817.05$

Example 4:

0.06 12

$3000 was invested at 6% per year compounded monthly.

a) Write the exponential function in the form: $A = P(1+i)^n$

$i = \frac{0.06}{12} = 0.005$

$A = 3000(1.005)^n$

b) What will be the future value of the investment after 4 years?

12 times/year

$A = 3000(1.005)^{48}$

$A = \$3811.47$

Example 5:

20%

0.20

An automobile that originally costs $24 000 loses one-fifth of its value each year. What is the value after 6 years?

$A = 24000(1-0.2)^6$

$A = 24000(0.8)^6$

$A = \$6291.46$

Example 6:

$2000 is invested for 3 years at an annual interest rate of 9% compounded monthly. Lucas solved the following equation:

$$A = 2000(1.0075)^3 \quad 36$$

Correct the error and solve the problem.

$A = 2000(1.0075)^{36}$

$A = \$2617.29$

Example 7:

Nora is about to invest $5000 in an account that pays 6% interest a year compounded monthly for the next 3 years. A different financial institution offers 6.5% interest a year compounded semi-annually for the next 3 years. Write a function that models the growth of Nora's investment for each situation. Should Nora invest her money in this financial institution instead? Explain why or why not.

$i = \frac{0.06}{12} = 0.005$

$n = 3 \times 12 = 36$

$A = 5000(1.005)^{36}$

$A = 5983.40$

$i = \frac{0.065}{2} = 0.0325$

$n = 3 \times 2 = 6$

$A = 5000(1.0325)^{6}$

$A = 6057.74$

Better Option!

Practice Questions:

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