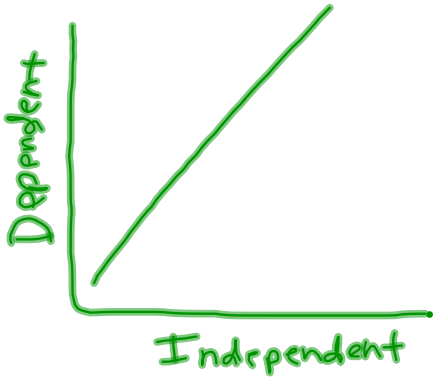
Section 5.6 - Properties of Linear Relations

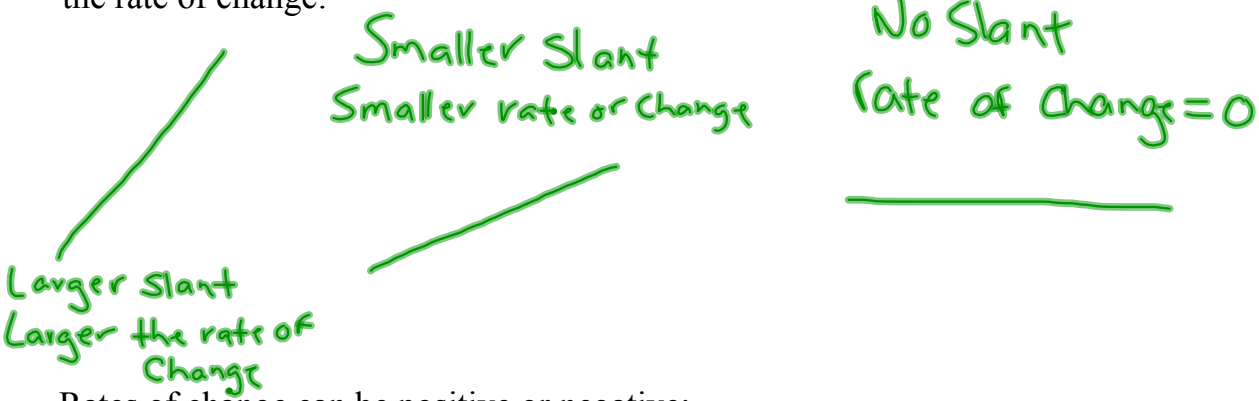
Rate of change:
The rate of change of a relation is defined as the slant, or the change in the dependent variable over the change in the independent variable.



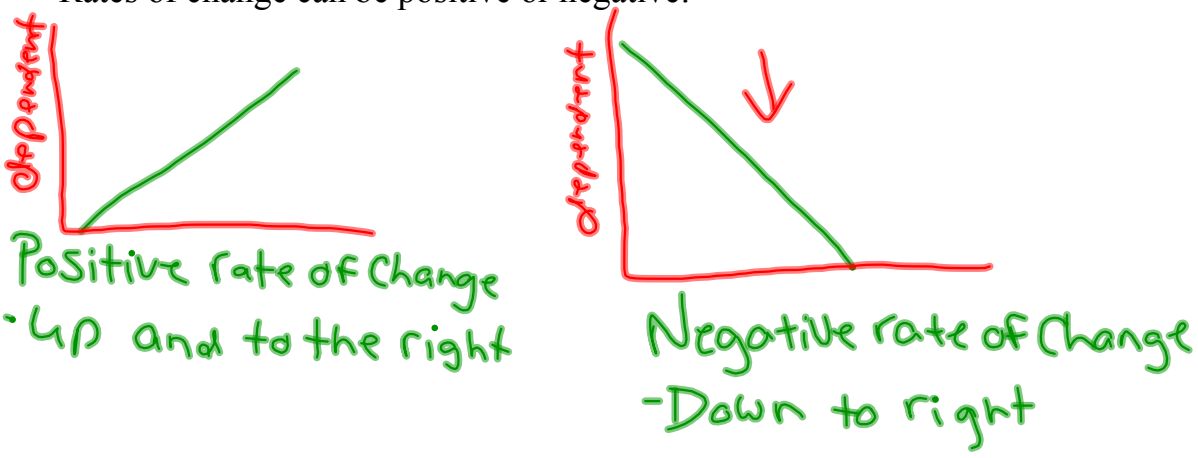
$$\text{Rate of Change} = \frac{\text{Change of Dependent}}{\text{Change of indepent.}}$$

$$= \frac{\text{Change in y-values}}{\text{Change in x-values}}$$

The greater the slant of a line on a graph, the greater the rate of change:



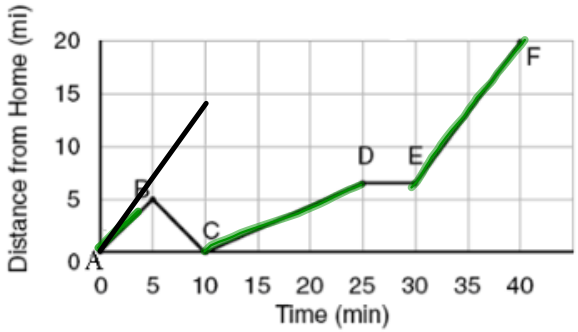
Rates of change can be positive or negative:



Applications of Rate of Change =

Sometimes we will be given a graph of distance vs. time. The rate of change of distance over time is called speed. Thus, we can comment on the speed of an object at different times by examining the slant of the graph.

The graph shows Mitchell leaving home at point A and going to a party at point F.



(i) What was the slowest speed? What was the maximum speed?

D-E - Stopped
C-D - Slowest Speed
E-D - Max Speed (Steepest Slant)

(ii) Which point represents when he turned around to go back home? What would be a reason for this?

B-C

(iii) When did Mitchell stop and why?

D-E

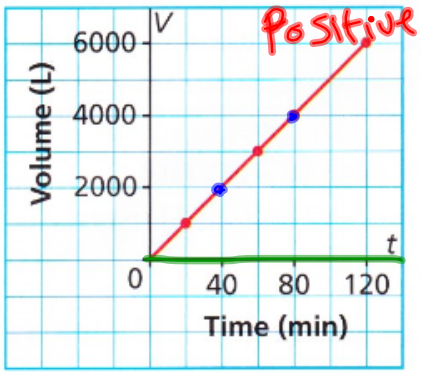
Calculating Rate of Change Steps:

•Pick any two points that lie on a line.
•Between the two points, determine the change in value of the dependent variable.
•Between the two points, determine the change in value of the independent variable.
•Calculate the rate of change by dividing the change in value of the dependent variable by the change in value of the independent variable.

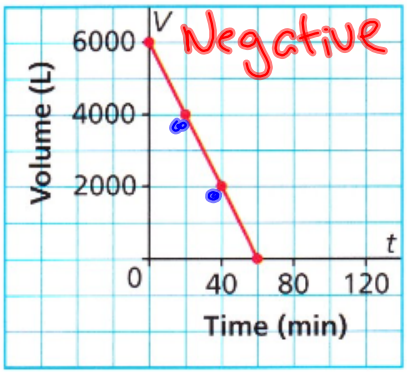
$$\text{Rate of Change} = \frac{\text{Change in dependent } (y)}{\text{Change in independent } (x)}$$

A water tank on a farm near Swift Current, Saskatchewan, holds 6000 L.
Graph A represents the tank being filled at a constant rate.
Graph B represents the tank being emptied at a constant rate.

Graph A
Filling a Water Tank



Graph B
Emptying a Water Tank



a) Identify the independent and dependent variables.

Time Volume

b) Determine the rate of change of each relation, then describe what it represents.

Points ① 40min, 2000L
② 80 min, 4000L

Rate of change $= \frac{4000L - 2000L}{80min - 40min}$

$= \frac{2000L}{40min}$

$= \frac{50L}{min}$

Ⓑ Points 40min, 2000L
20min 4000L

Rate of change $= \frac{4000L - 2000L}{20min - 40min}$

$= \frac{2000L}{-20min}$

$= -\frac{100L}{min}$

50L of water goes into the tank every minute

Rate of change from Table of values or ordered pairs:

The following table of values shows the cost of a pizza with up to five extra toppings.

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

1 < 12.00 → 0.75
1 < 12.75 → 0.75
1 < 13.50 → 0.75

a) Identify the Independent and Dependent Variables.

#of toppings Cost.

b) Determine the rate of change and what it represents.

$$\text{Rate of Change} = \frac{0.75\$}{1 \text{ topping}}$$

$$= \frac{0.75\$}{\text{Topping}}.$$

$0.75 per extra topping.

Homework:
Page 308 - # 7, 8, 11, 14, 17