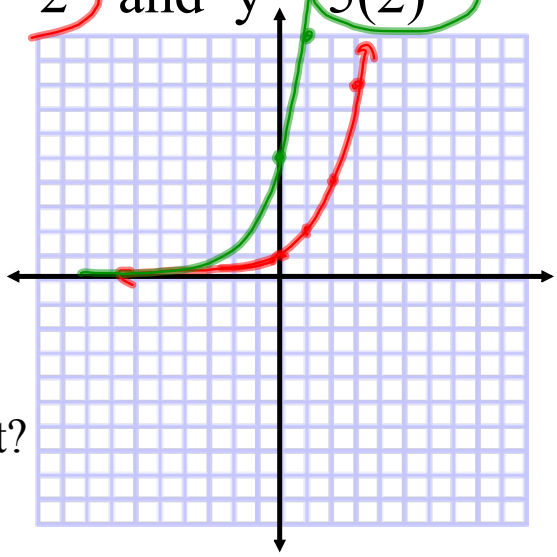
6.2 Relating Characteristics of an Exponential Function to its Equation

Exponential Function of the form: $y = a(b)^x$

How do the parameters of a and b affect the graph of the function? Compare : $y = 2^x$ and $y = 5(2)^x$

x	y
-2	
-1	
0	1
1	
2	

x	y
-2	
-1	
0	5
1	
2	



How is "a" related to the y-intercept?

How do we get the y-intercept algebraically? make $x = 0$

What happens because $b>1$? Growth

Why are there no x intercepts? can't get $y=0$

Does a or b affect the domain or range? No!

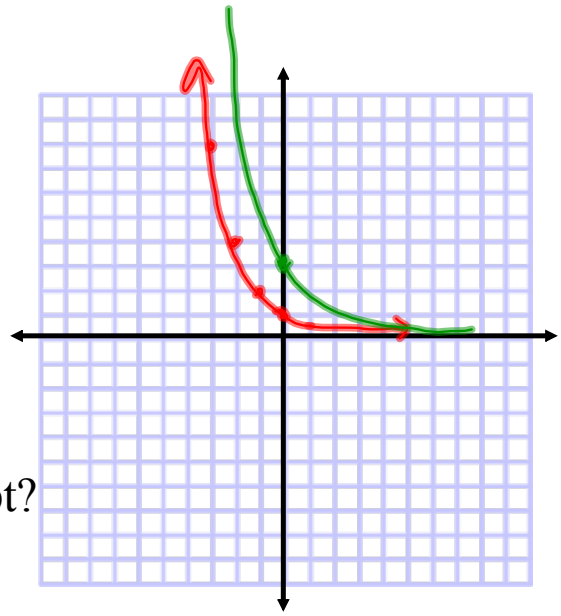
Domain $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y > 0, y \in \mathbb{R}\}$

Now Compare: $y = (\frac{1}{2})^x$ and $y = 3(\frac{1}{2})^x$

x	y
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

x	y
-2	12
-1	6
0	3
1	$\frac{3}{2}$
2	$\frac{3}{4}$



How is "a" related to the y-intercept?
How do we get the y-intercept algebraically? make x = 0
What happens because 0<b<1? Decay
Why are there no x intercepts? can't get y=0
Does a or b affect the domain or range? NO

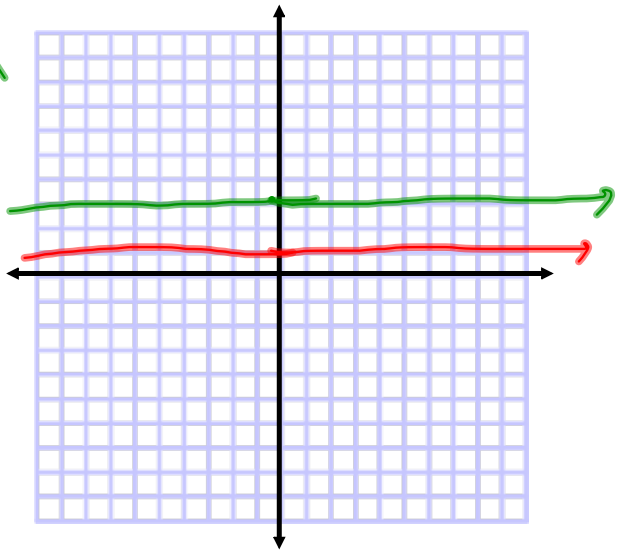
What would happen if b=1 or b<0?

Case 1: b = 1

$y = (1)^x$

$y = 3(1)^x$

horizontal line



x	y
-2	
-1	
0	
1	
2	

x	y
-2	
-1	
0	
1	
2	

b = 1 NO GOOD !

What type of graph do you get?

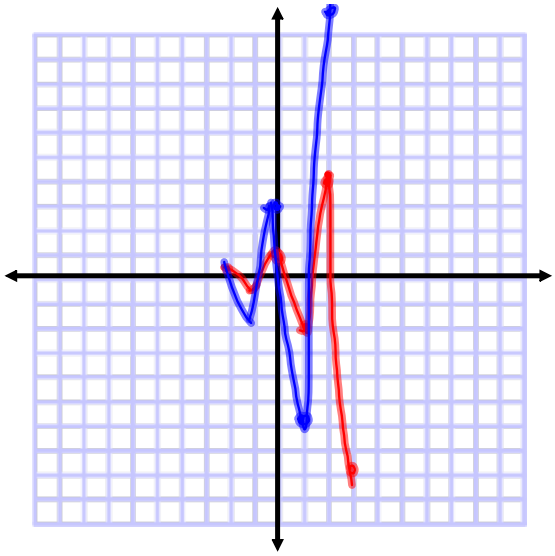
Case 2: $b < 0$

$y = (-2)^x$

$y = 3(-2)^x$

x	y
-2	$\frac{1}{4}$
-1	$-\frac{1}{2}$
0	1
1	-2
2	4

x	y
-2	
-1	
0	3
1	
2	



$b \leq 0$ NO GOOD!

If b is negative the values switch from positive to negative when integer values of x are chosen. If x was a fraction you might get non-real values: $(-2)^{1/2}$

How is $y = x^2$ different from $y = 2^x$? Relate the answer to domain/range and the shape of the graph.

$y = x^2$ parabola, neg values for x get bigger

$y = 2^x$ exponential, neg values for x get smaller

Ask students to respond to the following for $y = a(b)^x$ where $b > 0$, $b \neq 1$, and $a > 0$:

(i) Why does the function tend to zero as x gets very large when $0 < b < 1$? when base is a fraction between 0 and 1, bigger x values make the fraction smaller.

(ii) Why does the function go to ∞ as x goes to ∞ when $b > 1$?

(i) Ex $\left(\frac{1}{2}\right)^1 = \frac{1}{2}$ but $\left(\frac{1}{2}\right)^6 = \frac{1}{64}$ and $\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$

(ii) when base is bigger than 1, bigger x-values make the number go really large.

Ex: $3^1 = 3$, but $3^3 = 27$ and $3^5 = 243$

- Ask students to respond to the following for $y = a(b)^x$ where $b > 0$, $b \neq 1$, and $a > 0$:

  (i) Why does the function tend to zero as x gets very large when $0 < b < 1$?

  (ii) Why does the function go to ∞ as x goes to ∞ when $b > 1$?

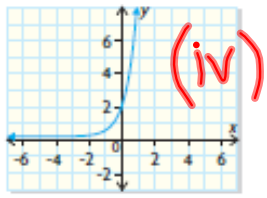
$y = 8\left(\frac{2}{3}\right)^x$	True	False	Why I think so
(i) the y-intercept is 1		✓	$a = 8$
(ii) the graph has one x-intercept		✓	Range is $y > 0$
(iii) the range is $\{y \mid y > 0, y \in R\}$	✓		
(iv) the domain is $\{x \mid x > 8, x \in R\}$		✓	$\{x \mid x \in \mathbb{R}\}$
(v) this is a decreasing exponential function	✓		$b = \frac{2}{3} < 1$

EXAMPLE 3 Matching an exponential equation with its corresponding graph

Which exponential function matches each graph below? Provide your reasoning.

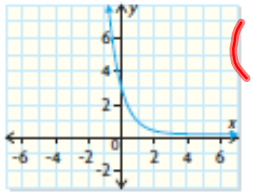
i) $y = 3(0.2)^x$ ii) $y = 4(3)^x$ iii) $y = 4(0.5)^x$ iv) $y = 2(4)^x$

a)



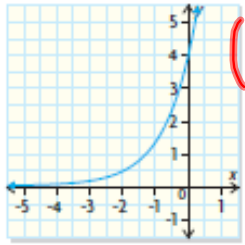
(iv)

c)



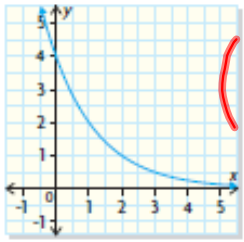
(i)

b)



(ii)

d)



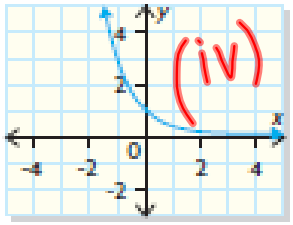
(iii)

Your Turn

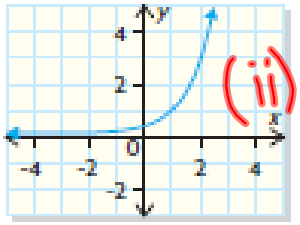
Match each function with the corresponding graph below. Provide your reasoning.

i) $y = (3)^x$ ii) $y = \frac{1}{3}(3)^x$ iii) $y = 3\left(\frac{1}{3}\right)^x$ iv) $y = \left(\frac{1}{3}\right)^x$

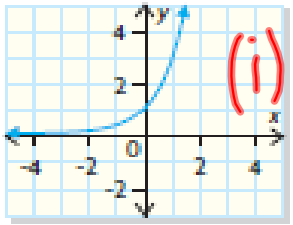
a)



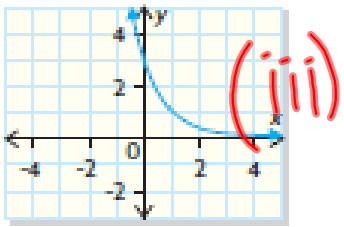
c)



b)



d)



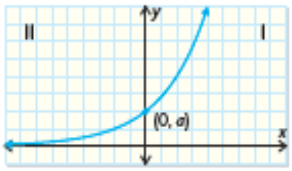
In Summary

Key Ideas

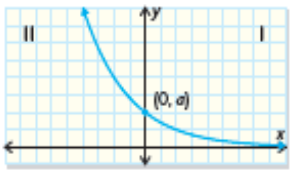
- In a table of values for an exponential function, there is a constant ratio between consecutive y-values when the x-values increase by the same amount. The value of this ratio is equal to the parameter b in the function $y = a(b)^x$, where $b \neq 1$.
- In an exponential function of the form $y = a(b)^x$, a is a non-zero multiplier and b is the base (where $b > 0$ and $b \neq 1$). The value of a is the y-intercept of the graph of the function.

Need to Know

- An exponential function is an increasing function if $a > 0$ and $b > 1$.
- An exponential function is a decreasing function if $a > 0$ and $0 < b < 1$.
- Changing the parameters a and b in exponential functions of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, does not change the number of x-intercepts, the end behaviour, the domain, or the range of the function. These characteristics are identical in all exponential functions of this form.



$a > 0, b > 1$



$a > 0, 0 < b < 1$

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