# 6.3 Solving Exponential Equations

Essentially two types:

1. Solving for unknown in exponent
2. Solving for unknown in base

**Type 1:** Note if $b^x = b^y$, then $x = y$

(If expressions are equal and bases are the same, then exponents must also be the same (equal)).

Ex: $2^x = 2^3$, then $x = 3$

Ex: $2^{2x+1} = 2^5 \rightarrow 2x+1 = 5$

$2x = 5-1$

$2x = 4$

$\boxed{x = 2}$

Ex③ $5^{2x-1} = 5^{4x-7}$, then $2x-1 = 4x-7$

$-1+7 = 4x-2x$

$6 = 2x$

$\boxed{3 = x}$

What if bases are not the same?

Ex: $4^x = 2^{x+1}$

Whenever possible, make them the same!

Ex④ $4^x = 2^{x+1}$

$(2^2)^x = 2^{x+1}$

$2^{2x} = 2^{x+1}$

$2x = x+1$

$2x - x = 1$

$x = 1$

Ex⑤ $9^{4x} = 81$

$9^{4x} = 9^2$

$4x = 2$

$\frac{4x}{4} = \frac{2}{4}$

$x = \frac{1}{2}$

Ex 6

$8^{x-1} = 2^{x+1}$

$(2^3)^{(x-1)} = 2^{x+1}$

$2^{3x-3} = 2^{x+1}$

$3x-3 = x+1$

$3x-x = 1+3$

$2x = 4$

$\boxed{x=2}$

Ex 7:

$8^x = 2^7 \cdot 4^9$

$8^x = 2^7 \cdot (2^2)^9$

$8^x = 2^7 \cdot 2^{18}$

$8^x = 2^{7+18}$

$(2^3)^x = 2^{25}$

$2^{3x} = 2^{25}$

so, $3x = 25$

$\boxed{x = \frac{25}{3}}$

Ex 8

$27^{1-x} = \left(\frac{1}{9}\right)^{2-x}$

$(3^3)^{(1-x)} = \left(\frac{1}{3^2}\right)^{2-x}$

$3^{3-3x} = (3^{-2})^{(2-x)}$

$3^{3-3x} = 3^{-4+2x}$

$3-3x = -4+2x$

$3+4 = 2x+3x$

$\frac{7}{5} = \frac{5x}{5}$

$\boxed{x = \frac{7}{5}}$

Ex 9

$8^x = \sqrt[3]{16}$

$8^x = 16^{\frac{1}{3}}$

$(2^3)^x = (2^4)^{\frac{1}{3}}$

$2^{3x} = 2^{\frac{4}{3}}$

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$

$\frac{3x}{1} = \frac{4}{3}$

$\frac{9x}{9} = \frac{4}{9}$

$\boxed{x = \frac{4}{9}}$

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4 (e)

$2^{x+2} = (8)^{\frac{1}{2}}$

$2^{x+2} = (2^3)^{\frac{1}{2}}$

$2^{x+2} = 2^{\frac{3}{2}}$

$x+2 = \frac{3}{2}$

$x = \frac{3}{2} - \frac{2}{1}\frac{2}{2}$

$x = \frac{3}{2} - \frac{4}{2}$

$\boxed{x = -\frac{1}{2}}$

7. (b)

$\frac{\cancel{8} \cdot (3)^{\frac{x}{2}}}{\cancel{8}} = \frac{72}{8}$

$(3)^{\frac{x}{2}} = 9$

$(3)^{\frac{x}{2}} = 3^2$

$\cancel{2}\left(\frac{x}{\cancel{2}}\right) = (2)^2$

$\boxed{x = 4}$

#10

$$\frac{2^x}{2^x} = 2^{-2x}$$

$$2^{x-x} = 2^{-2x}$$

$$2^0 = 2^{-2x}$$

$$0 = -2x$$

$$x = 0$$

11.

$$10^{-3x} \cdot 10^x = \frac{1}{10}$$

$$10^{-3x+x} = \frac{1}{10}$$

$$10^{-2x} = 10^{-1}$$

$$\frac{\cancel{-2}x}{\cancel{-2}} = \frac{-1}{-2}$$

$$x = \frac{1}{2}$$

# Problem Solving with Exponentials

A very common problem involves half-life.

Half-life → amount of time for some material/element to decay to half of its original size.

Half-life functions usually look like,

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$ where

$A_0$ = original amount/mass
$A$ = final amount/mass
$h$ = the elements half-life
$t$ = time

Ex: ① A certain material has a half-life of 100 days. If there were 10g present initially, how much will remain after 60 days? Round your answer to the nearest hundredth of a gram.

$A_0 = 10g$
$h = 100$ days
$t = 60$ days
$A = ?$

Solution:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{100}} = 10(0.5)^{0.6}$$

$$A = 10\left(\frac{1}{2}\right)^{\frac{60}{100}}$$

$$A = 6.60\,g$$

Ex: ② The half-life of a radioactive isotope is 30 hours. ~~How long~~ will it take a sample of 1792 mg to decay to 56 mg?

$h = 30$ hrs
$A_0 = 1792$ mg
$A = 56$ mg
$t = ?$

Solution: $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{30}}$

$$\frac{56}{1792} = \frac{\cancel{1792}\left(\frac{1}{2}\right)^{\frac{t}{30}}}{\cancel{1792}}$$

$$\frac{1}{32} = \left(\frac{1}{2}\right)^{\frac{t}{30}}$$

$$\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{\frac{t}{30}}$$

$$30(5) = \left(\frac{t}{\cancel{30}}\right)\cancel{30}$$

$$150 = t$$

It take 150 hrs

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#10. $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{5.3}}$

(a) half-life = 5.3

(b) what percent remains after 10 yrs?

$A = 100\left(\frac{1}{2}\right)^{\frac{10}{5.3}}$

$A = 27.04\%$

$A = 30\left(\frac{1}{2}\right)^{\frac{10}{5.3}}$

$A = 8.11$

$\frac{8.11}{30} \times 100\%$

$= \boxed{27\%}$

(c) only 10% remaining:

$10 = 100\left(\frac{1}{2}\right)^{\frac{t}{5.3}}$ graph

$\frac{1}{10} = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$

$t = 17.606219$ yrs

15. $P(h) = 300(2)^{\frac{h}{12}}$ $A = A_0(2)^{\frac{t}{d}}$

$P(h) = 307200$ , $h = ?$

$$\frac{307200}{300} = \frac{\cancel{300}(2)^{\frac{h}{12}}}{\cancel{300}}$$

$$1024 = 2^{\frac{h}{12}}$$

$$2^{10} = 2^{\frac{h}{12}}$$

$$\frac{10}{1} = \frac{h}{12}$$

$$\boxed{h = 120}$$

| x | y |
|---|---|
| 0 | 3 |
| 1 | 15 |
| 2 | 75 |
| 3 | 375 |
| 4 | 1875 |
| 5 | 9375 |

y-int

$y = a(b)^x$

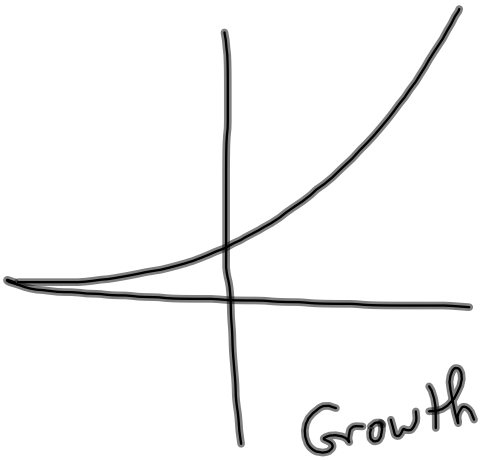
common ratio

$> 5$

$\frac{75}{15} = 5$ $\frac{375}{75} = 5$

$\frac{1875}{375} = 5$

So $y = 3(5)^x$



Growth
Increasing

Decay
Decreasing

Extra Practice/Review
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7, 11 a, b, c