# Math 3201 Midterm Exam Review

## Name: \_\_\_\_\_

#### **Multiple Choice**

Identify the choice that best completes the statement or answers the question.

- **1.** What is the meaning of *complement* in set theory?
  - A. all the elements in the universal set that are not identical
  - **B.** a set of elements that work well with a given set
  - C. all the elements of a universal set that do not belong to a subset of it
  - **D.** all the elements that are the opposite of the elements in a given set
- **2.** What is the meaning of *disjoint* in set theory?
  - A. two or more sets having no elements in common
  - **B.** two or more sets that do not match
  - C. sets that are in different universal sets
  - **D.** sets that contain no elements
  - **3.** What is the universal set?
    - A. a set with an infinite number of elements
    - B. a set of all the elements under consideration for a particular context
    - C. a set with a countable number of elements
    - **D.** a set that contains every possible element
    - **4.** Which statement is true?
      - A. The English language and the French language are disjoint sets.
      - **B.** Hockey equipment and lacrosse equipment are disjoint sets.
      - C. Band instruments and orchestral instruments are disjoint sets.
      - **D.** Linear equations and quadratic equations are disjoint sets.
      - 5. Which pair of sets represents disjoint sets?
        - A. *N*, the set of natural numbers, and *I*, the set of integers
        - **B.** *T*, the set of all triangles, and *C*, the set of all circles
        - C. *N*, the set of natural numbers, and *P*, the set of positive integers
        - **D.** none of the above
      - **6.** Rahim described the set as follows:
        - $M = \{ all of the foods he eats \}$
        - $D = \{$ his favourite desserts $\}$
        - *V* = {his favourite vegetables}
        - $F = \{$ his favourite fruits $\}$

Which are the disjoint sets?

- **A.** *M* and *D* **B.** *M* and *V* **C.** *M* and *F* **D.** *V* and *F*
- **7.** Given the following situation:
  - the universal set  $U = \{ \text{positive integers less than } 20 \}$
  - $X = \{4, 5, 6, 7, 8\}$
  - $P = \{ \text{prime numbers of } U \}$
  - $O = \{ \text{odd numbers of } U \}$

Which set represents the odd, prime numbers of set U?

- A. {0, 3, 5, 7, 11, 13, 17, 19}
  B. {3, 5, 7, 11, 13, 17, 19}
  C. {2, 3, 5, 7, 11, 13, 17, 19}
  D. {1, 2, 3, 5, 7, 11, 13, 17, 19}
- There are 28 students in Mr. Connelly's Grade 12 mathematics class.
   The number of students in the yearbook club and the number of students on student council are shown in the Venn diagram. Use the diagram to answer the following questions.



How many students are in both the yearbook club and on the student council?

- A. 2B. 5
- **C.** 1
- **D**. 7
- 9. There are 28 students in Mr. Connelly's Grade 12 mathematics class.

The number of students in the yearbook club and the number of students on student council are shown in the Venn diagram. Use the diagram to answer the following questions.



How many students are in at least one of the yearbook club or on student council?

- **A.** 2
- **B.** 5
- **C.** 8
- **D.** 7

**10.** Consider the following Venn diagram of herbivores and carnivores:



Determine  $n(H \cup C)$ .

- **A.** 2
- **B.** 9
- C. 4D. 3
- **11.** Consider the following Venn diagram of herbivores and carnivores:



Determine  $H \cap C$ .

A. {moose, rabbit, deer, squirrel}

- **B.** {bear, raccoon, badger}
- **C.** {cougar, wolf}
- **D.** {moose, rabbit, deer, squirrel, bear, raccoon, badger, cougar, wolf}
- **12.** Consider the following two sets:
  - $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
  - $B = \{-9, -6, -3, 0, 3, 6, 9, 12\}$

Which Venn diagram correctly represents these two sets?



**13.** A summer camp offers canoeing, rock climbing, and archery. The following Venn diagram shows the types of activities the campers like.



Use the diagram to determine  $n((R \cup C) \setminus A)$ .

**A.** 64

**B.** 48

- **C.** 37
- **D.** 59
- 14. A restaurant offers Chinese, Thai, and Korean food. The following Venn diagram shows the types of food the customers like.



Use the diagram to determine  $n(K \setminus T)$ .

**A.** 10

- **B.** 19
- **C.** 50
- **D.** 23

**15.** Eve can choose from the following notebooks:

- lined pages come in red, green, blue, and purple
- graph paper comes in orange and black

If Eve needs one lined notebook and one with graph paper, which of the following pairs is not a possible outcome?

- A. red and orange
- **B.** black and blue
- C. green and red
- **D.** purple and black

- **16.** A combination lock opens with the correct four-letter code. Each wheel rotates through the letters A to L. How many different four-letter codes are possible?
  - **A.** 20 736
  - **B.** 48
  - **C.** 1728
  - **D.** 456 976
- **17.** A restaurant offers 60 flavours of wings and your choice of three dips. How many variations of wings and dip can you order?
  - **A.** 20
  - **B.** 60
  - **C.** 180
  - **D.** 216 000
- **18.** How many possible ways can you draw a single card from a standard deck and get either a heart or a club?
  - **A.** 2
  - **B.** 13
  - **C.** 14
  - **D.** 26
- \_\_\_\_ **19.** Evaluate. 8! + 1!
  - A. 40 321
    B. 5041
    C. 40 123
    D. 16 777 217
- **20.** Evaluate.  $(3!)^2$ 
  - A. 8B. 9C. 18
  - **D.** 36
- \_\_\_\_ **21.** Evaluate.
  - <u>41-71</u> 81
  - **A.** 0
  - **B.** 1
  - **C.** 3

- **D.**  $\frac{1}{3}$
- $\underline{\qquad 22. Evaluate.}_{\underline{1001!}}$

999!

- **A.** 1 000 000
- **B.** 1 001 000
- **C.** 10 100 100
- **D.** 999 999

**23.** Identify the expression that is equivalent to the following:

$$\frac{n!}{(n-2)!} + n$$

- **A.** n **B.** -n **C.**  $n^2$ **D.**  $n^3$
- **24.** Solve for *n*, where  $n \in I$ .

$$\frac{n!}{(n-1)!} = 4!$$

- **A.** 8**B.** 16**C.** 24
- **D.** 32
- 25. Solve for *n*, where *n* ∈ I.  $\frac{(n-2)!}{(n-3)!} = 15$ 
  - (n 3)!
  - A. 13
    B. 15
    C. 17
    D. 18

26. How many different permutations can be created when 7 people line up to buy movie tickets?

- **A.** 49 **B.** 128 **C.** 720 **D.** 5040
- **27.** How many different permutations can be created when Anneliese, Becky, Carlo, Dan, and Esi line up to buy movie tickets, if Esi always stands immediately behind Becky?

- **A.** 48
- **B.** 120**C.** 720
- **D.** 24

**28.** Evaluate.

- $_{3}P_{1}$
- **A.** 1
- **B.** 2
- **C.** 3
- **D.** 6

**29.** Evaluate.

- $_{21}P_{2}$
- **A.** 441**B.** 420
- **C.** 399
- **D.** 2 097 152
- **30.** Suppose a word is any string of letters. How many two-letter words can you make from the letters in LETHBRIDGE if you do not repeat any letters in the word?
  - **A.** 72**B.** 100**C.** 81
  - **D.** 90
  - **31.** Solve for *n*.  ${}_{n}P_{4} = 120$ 
    - **A.** n = 5
    - **B.** *n* = 6
    - **C.** n = 7
    - **D.** *n* = 8
- **32.** How many ways can 7 friends stand in a row for a photograph if Sheng always stands beside his girlfriend?
  - **A.** 1440
  - **B.** 5040
  - **C.** 360
  - **D.** 720
  - **33.** How many ways can 8 friends stand in a row for a photograph if Molly, Krysta, and Simone always stand together?

- **A.** 1440**B.** 4320
- **C.** 5040
- **D.** 2160

\_\_\_\_ **34.** Evaluate.

- 61 21-21
- **A.** 48
- **B.** 72
- **C.** 140
- **D.** 180

\_\_\_\_ 35. How many different arrangements can be made using all the letters in CANADA?

- **A.** 120
- **B.** 180
- **C.** 360
- **D.** 720

**36.** How many different routes are there from A to B, if you only travel south or east?



- **A.** 16
- **B.** 24
- **C.** 28
- **D.** 56
- **37.** Eight quarters are flipped simultaneously. How many ways can three coins land heads and five coins land tails?
  - **A.** 36
  - **B.** 42
  - **C.** 50
  - **D.** 56
- **38.** There are 14 members of a student council. How many ways can 4 of the members be chosen to serve on the dance committee?

- **A.** 1001
- **B.** 2002
- **C.** 6006
- **D.** 24 024
- **39.** A fun fair requires 4 employees to work at the sack bar. There are 13 people available. How many ways can a group of 4 be chosen?
  - **A.** 1000
  - **B.** 715
  - **C.** 635
  - **D.** 808
- **40.** Evaluate.
  - $\binom{9}{4}$
  - **A.** 130
  - **B.** 126
  - **C.** 122
  - **D.** 118

**41.** How many ways can 4 representatives be chosen from a hockey team of 17 players?

- **A.** 2380
- **B.** 57 120
- **C.** 31 060
- **D.** 9575

**42.** Which of the following is equivalent to  ${}_{18}C_{15}$ ?

- A.  $\frac{18!}{3!}$ B.  $\frac{18!}{3 \cdot 15!}$ C.  $\frac{18!}{3! \cdot 15!}$ D.  $\frac{18!}{15!}$
- **43.** Given the following probabilities, which event is most likely to occur?
  - **A.** P(A) = 0.2 **B.**  $P(B) = \frac{1}{6}$ **C.** P(C) = 0.3

- **D.**  $P(D) = \frac{1}{3}$
- **44.** Three events, *A*, *B*, and *C*, are all equally likely. If there are no other possible events, which of the following statements is true?
  - **A.** P(A) = 0 **B.**  $P(B) = \frac{1}{3}$  **C.** P(C) = 1**D.** P(A) = 3
- **45.** Raymond has 12 coins in his pocket, and 9 of these coins are quarters. He reaches into his pocket and pulls out a coin at random. Determine the odds against the coin being a quarter.
  - A. 1:4
    B. 1:3
    C. 3:4
    D. 3:1
- **46.** Tia notices that yogurt is on sale at a local grocery store. The last eight times that yogurt was on sale, it was available only three times. Determine the odds against yogurt being available this time.
  - **A.** 3:5
  - **B.** 3:8
  - **C.** 5:8
  - **D.** 5:3
- **47.** Zahra likes to go rock climbing with her friends. In the past, Zahra has climbed to the top of the wall 7 times in 28 attempts. Determine the odds against Zahra climbing to the top.
  - **A.** 3:1 **B.** 4:1
  - **C.** 3:11
  - **D.** 3:4
- **48.** Julie draws a card at random from a standard deck of 52 playing cards. Determine the probability of the card being a diamond.
  - **A.** 0.250
  - **B.** 0.500
  - **C.** 0.625
  - **D.** 0.750
- **49.** The weather forecaster says that there is an 80% probability of rain tomorrow. Determine the odds against rain.

**A.** 4:5

- **B.** 4:1
- **C.** 1:5
- **D.** 1:4
- **50.** The weather forecaster says that there is a 50% probability of showers tomorrow. Determine the odds against showers.
  - **A.** 1:1
  - **B.** 5 : 10
  - **C.** 2:1
  - **D.** 1:2
- **51.** A sports forecaster says that there is a 40% probability of a team winning their next game. Determine the odds against that team winning their next game.
  - A. 2:3
    B. 2:5
    C. 3:5
  - **D.** 3:2
- **52.** Nine boys and twelve girls have signed up for a trip. Only six students will be selected to go on the trip. Determine the probability that only boys will be on the trip.
  - **A.** 0.02%
  - **B.** 0.08%
  - **C.** 0.15%
  - **D.** 0.23%
- **53.** Four boys and three girls will be riding in a van. Only two people will be selected to sit at the front of the van. Determine the probability that only boys will be sitting at the front.
  - **A.** 28.57%
  - **B.** 33.45%
  - **C.** 39.06%
  - **D.** 46.91%
- \_\_\_\_\_ 54. Cai tosses four coins. Determine the probability that they all land as tails.
  - **A.** 6.25%
  - **B.** 12.50%
  - **C.** 18.75%
  - **D.** 25.00%
  - **\_ 55.** Select the events that are mutually exclusive.
    - A. Drawing a 7 or drawing a heart from a standard deck of 52 playing cards.
    - **B.** Rolling a sum of 4 or rolling an even number with a pair of four-sided dice, numbered 1 to 4.

- **C.** Drawing a black card or drawing a Queen from a standard deck of 52 playing cards.
- **D.** Rolling a sum of 8 or a sum of 11 with a pair of six-sided dice, numbered 1 to 6.
- **56.** Select the events that are mutually exclusive.
  - **A.** Drawing a red card or drawing a diamond from a standard deck of 52 playing cards.
  - **B.** Rolling a sum of 8 or rolling an even number with a pair of six-sided dice, numbered 1 to 6.
  - **C.** Drawing a black card or drawing a Queen from a standard deck of 52 playing cards.
  - **D.** Drawing a 3 or drawing an even card from a standard deck of 52 playing cards.
- 57. Roena is about to draw a card at random from a standard deck of 52 playing cards. Determine the probability that she will draw a heart or a King.
  - A. 13
  - **B**.
  - 9 26
  - **C.** 11
  - 18
  - **D.** 11 12
  - **58.** Select the events that are dependent.
    - **A.** Drawing a face card from a standard deck of 52 playing cards, putting it back, and then drawing another face card.
    - **B.** Rolling a 4 and rolling a 3 with a pair of six-sided dice, numbered 1 to 6.
    - **C.** Drawing a heart from a standard deck of 52 playing cards, putting it back, and then drawing another heart.
    - **D.** Rolling a 3 and having a sum greater than 5 with a pair of six-sided dice, numbered 1 to 6.
  - **59.** Select the events that are dependent.
    - **A.** Rolling a 2 and rolling a 5 with a pair of six-sided dice, numbered 1 to 6.
    - **B.** Drawing an odd card from a standard deck of 52 playing cards, putting it back, and then drawing another odd card.
    - C. Drawing a spade from a standard deck of 52 playing cards and then drawing another spade, without replacing the first card.
    - **D.** Rolling an even number and rolling an odd number with a pair of six-sided dice, numbered 1 to 6.

- **60.** Rino has six loonies, four toonies, and two quarters in his pocket. He needs two loonies for a parking meter. He reaches into his pocket and pulls out two coins at random. Determine the probability that both coins are loonies.
  - **A.** 16.3%
  - **B.** 18.4%
  - **C.** 22.7%
  - **D.** 25.9%
- **61.** Carlo goes to the gym and does two different cardio workouts each day. His choices include using a treadmill, a stationary bike, and running the track. Determine the probability that the next time Carlo goes to the gym will use the stationary bike and then run the track.
  - **A.** 16.7%
  - **B.** 26.1%
  - **C.** 33.4%
  - **D.** 41.9%
- **62.** Select the independent events.
  - A. P(A) = 0.22, P(B) = 0.39, and  $P(A \cap B) = 0.072$
  - **B.** P(A) = 0.18, P(B) = 0.7, and  $P(A \cap B) = 0.163$
  - C.  $P(A) = 0.51, P(B) = 0.1, \text{ and } P(A \cap B) = 0.069$
  - **D.**  $P(A) = 0.9, P(B) = 0.23, \text{ and } P(A \cap B) = 0.207$
- **63.** Select the independent events.
  - A. P(A) = 0.67, P(B) = 0.12, and  $P(A \cap B) = 0.086$
  - **B.** P(A) = 0.83, P(B) = 0.4, and  $P(A \cap B) = 0.378$
  - C. P(A) = 0.4, P(B) = 0.91, and  $P(A \cap B) = 0.364$
  - **D.** P(A) = 0.2, P(B) = 0.32, and  $P(A \cap B) = 0.046$

**64.** Identify the rational expression that is equivalent to  $\frac{2-x}{3x}$ .

A.  $\frac{4-2x}{3x}$ <br/>B.  $\frac{4-2x}{6x}$ <br/>C.  $\frac{2-2x}{6x}$ <br/>D.  $6x - 3x^2$ 

**65.** Identify the rational expression that is equivalent to  $\frac{4x}{x+5}$ .

A. 
$$\frac{x}{x+5}$$
  
B. 
$$\frac{4x^2}{x+5x^2}$$
  
C. 
$$\frac{2x}{0.5x+2.5}$$
  
D. 
$$\frac{4x(x+5)}{x+5}$$

- **66.** Determine the non-permissible value(s) for  $\frac{2-x}{3x}$ .
  - **A.**  $x \neq 2$  **B.**  $x > 0, x \neq 2$  **C.**  $x \neq 0$ **D.**  $x \ge 0$

**67.** Determine the non-permissible value(s) for  $\frac{1-x^2}{6}$ .

- **A.**  $x \neq 0$ **B.** x > 0
- **D.**  $x \neq 0$ **C.**  $x \neq 1$
- **D.** no restrictions

**68.** Determine the non-permissible value(s) for  $\frac{9-3x}{12x^2+x}$ .

A. 
$$x \neq 0, x \neq -\frac{1}{12}$$
  
B.  $x \neq 0$   
C.  $x \neq -\frac{1}{12}$   
D.  $x \neq 0, x \neq \frac{1}{12}$ 

**69.** Determine the non-permissible value(s) for  $\frac{3x + 2x^2}{x^2 - 6x}$ .

- A.  $x \neq 0, x \neq \frac{1}{6}$ B.  $x \neq 0, x \neq 6$ C.  $x \neq 6$ D.  $x \neq 0, x \neq -6$
- **70.** Which rational expression is simplified?

i) 
$$\frac{8x + 14}{x + 3x^2}$$
  
ii)  $\frac{7x^3}{10x^2 + 30x}$   
iii)  $\frac{11x}{x^2 + 1}$   
A. i)  
B. ii) and iii)  
C. iii)  
D. i) and iii)

71. Determine the non-permissible values for the rational expression  $\frac{2x}{6x-4x^2}$ .

A. 
$$x \neq 0, x \neq -\frac{3}{2}$$
  
B.  $x \neq 0$   
C.  $x \neq 0, x \neq \sqrt{\frac{3}{2}}$   
D.  $x \neq 0, x \neq \frac{3}{2}$ 

**72.** Determine the non-permissible values for the rational expression  $\frac{3x+6}{21x}$ .

A.  $x \neq 0, x \neq 7$ B.  $x \neq 0$ C.  $x \neq 0, x \neq 21$ D.  $x \neq 0, x \neq 2$ 

**73.** Simplify  $\frac{-21k}{14k^3}$ .

A. 
$$\frac{-3}{2k^2}, k \neq 0$$
  
B. 
$$\frac{-3k}{2k^2}, k \neq 0$$
  
C. 
$$\frac{-7}{k^2}, k \neq 0$$
  
D. 
$$\frac{-21}{14k^2}, k \neq 0$$

\_\_\_\_ **74.** Simplify  $\frac{81s^2}{36s}$ .

**A.** 
$$\frac{3s}{2}, s \neq 0$$

B. 
$$\frac{9s^2}{12s}, s \neq 0$$
  
C.  $\frac{9s}{4}, s \neq 0$   
D.  $\frac{27s}{12}, s \neq 0$   
75. Simplify  $\frac{16h^2 - 40h}{24h^3}$ .  
A.  $\frac{h-5}{3h^2}, h \neq 0$   
B.  $\frac{2h-5}{3h^2}, h \neq 0$   
C.  $\frac{2h-10}{6h^2}, h \neq 0$   
D.  $\frac{2h-5}{3h}, h \neq 0$   
76. Simplify  $\frac{6a+54a^2}{18-24a}$ .  
A.  $\frac{1+9a}{3-4a}, a \neq \frac{3}{4}$   
B.  $\frac{2a+27a^2}{9-12a}, a \neq \frac{3}{4}$   
C.  $\frac{3a+18a^2}{6-8a}, a \neq \frac{3}{4}$   
D.  $\frac{a+9a^2}{3-4a}, a \neq \frac{3}{4}$ 

\_\_\_\_\_ **77.** Determine all the non-permissible values of the variable.

$$\frac{3n-1}{n+5} \cdot \frac{2n}{2-n}$$
**A.**  $n \neq -5, -2, 0$ 
**B.**  $n \neq -5, 2$ 
**C.**  $n \neq 0$ 
**D.**  $n \neq -5, 2, 0$ 

78. Simplify 
$$\frac{3p^2}{2p(1+p)} \cdot \frac{p+4}{6}$$
.

\_\_\_\_\_

A. 
$$\frac{p^2 + 1}{1 + p}, p \neq 0, -1$$
  
B.  $\frac{p^2 + 4p}{4 + 4p}, p \neq 0, -1$   
C.  $\frac{1}{4}, p \neq 0, -1$   
D.  $\frac{p + 4}{4 + 4p}, p \neq 0, -1$ 

**79.** Simplify 
$$\frac{6d+3}{3d} \div \frac{2d+1}{4}$$
.

A. 
$$\frac{1}{2d}, d \neq 0, -\frac{1}{2}$$
  
B.  $\frac{4}{3d}, d \neq 0, -\frac{1}{2}$   
C.  $\frac{3}{d+1}, d \neq 0, -\frac{1}{2}$   
D.  $\frac{4}{d}, d \neq 0, -\frac{1}{2}$ 

80. Determine all the non-permissible values of the variable.  $\frac{3}{5g} \div \frac{5g-15}{2g+1}$ 

 $\frac{3}{5g} \div \frac{5g-15}{2g+1}$  **A.**  $g \neq 0, \frac{1}{2}$  **B.**  $g \neq 0, \frac{1}{2}, 3$  **C.**  $g \neq 0, -\frac{1}{2}$  **D.**  $g \neq 0, -\frac{1}{2}, 3$ 

**81.** Determine the lowest common denominator for the pair  $\frac{a}{3a+2}$  and  $\frac{80}{3a^2}$ .

**A.**  $9a^3 + 6a^2$  **B.**  $3a^3 + 2a^2$  **C.**  $6a^3$ **D.**  $3a^2$ 

**82.** Determine the lowest common denominator for the pair  $\frac{c}{3c^2 - 3}$  and  $\frac{3}{c+1}$ .

**A.**  $3c^3 + c - 2$  **B.**  $c^2 - 1$  **C.**  $3c^2 - 3$ **D.** c + 1

**83.** Determine all the non-permissible values of the variable.

$$\frac{1}{9-g} - \frac{g+1}{g^2-9}$$
**A.**  $g \neq -1, -3, 3, 9$ 
**B.**  $g \neq -1, 9$ 
**C.**  $g \neq 3, 9$ 
**D.**  $g \neq -3, 3, 9$ 

**84.** Determine all the non-permissible values of the variable.

$$\frac{4}{k+2} + \frac{8k}{k-8}$$
A.  $k \neq 0, -2, 8$ 
B.  $k \neq 0, 2, -8$ 
C.  $k \neq -2, 8$ 
D.  $k \neq 2, -8$ 
85. Simplify  $\frac{2u+10}{2u^2} - \frac{3}{u^2}$ .
A.  $\frac{2u+13}{2u^2}, u \neq 0$ 
B.  $\frac{2u-13}{2u^2}, u \neq 0$ 
C.  $\frac{u+4}{u^2}, u \neq 0$ 
D.  $\frac{u+2}{u^2}, u \neq 0$ 
86. Simplify  $\frac{3w}{12} - \frac{12}{5w}$ .
A.  $\frac{3w-12}{12-5w}, w \neq 0$ 
B.  $\frac{36w}{12-5w}, w \neq 0$ 
C.  $\frac{5w^2-48}{20w}, w \neq 0$ 

**D.** 
$$\frac{5w^2 - 144}{20w}, w \neq 0$$

- 87. Solve the following equation for x.  $\frac{2}{x} + \frac{1}{x+1} = \frac{7}{2x+2}$ A. x = 2B. x = 3
  - **C.** x = 4**D.** x = 5
- **88.** Solve the following equation for *x*.
  - $\frac{3x}{x+1} = 2$
  - **A.** x = 2 **B.** x = 3 **C.** x = 4**D.** x = 5
  - **89.** Identify the root(s) of the following equation.
    - $\frac{1}{4-x} = \frac{x}{4}$
    - **A.** x = 2 **B.** x = 3 **C.** x = 4**D.** A and B

## Short Answer

- 1. What is the set notation for the set of all positive real numbers that are less than 22?
- 2. What is the set notation for the set of all integers from -21 to -4 that are a multiple of 2?
- 3. Carlos surveyed 50 students about their favourite subjects in school. He recorded his results.

Favourite Subject	Number of Students
mathematics	18
science	15
neither mathematics nor science	20

Determine how many students like only mathematics or only science.

**4.** Mrs. Lam's physics class is visiting the local amusement park. She has 32 students. Of these students, 20 plan to ride the roller coaster and 15 plan to ride the vertical drop. There are 8 students who do not plan to ride either attraction.

Determine how many students plan to ride both the roller coaster and the vertical drop.

- 5. Grade 12 students were surveyed about their extra curricular activities.
  - 58% belonged to a sports team (*S*)
  - 63% belonged to a band or choir (*B*)
  - 47% belonged to a school club (C)
  - 24% belonged to a sports team and a band or choir
  - 21% belonged to a sports team and a school club
  - 36% belonged to a band or choir and a school club
  - 19% engaged in all three activities



What percent of students only belong to a band or choir? Write your answer in set notation.

- 6. The city surveyed 3000 people about how they travel to work.
  - 1978 took public transit (*P*)
  - 1494 drove (*D*)
  - 818 cycled (*C*)
  - 731 took public transit and drove only
  - 298 took public transit and cycled only
  - 27 drove and cycled only
  - 164 used all three modes of transportation

How many people use public transit only? Use a Venn diagram to show your answer.

- 7. The "Pita Patrol" offers these choices for each sandwich:
  - white or whole wheat pitas
  - 3 types of cheese
  - 5 types of filling
  - 12 different toppings
  - 4 types of sauce

How many different pitas can be made with 1 cheese, 1 filling, 1 topping, and no sauce?

8. Evaluate.

- <u>5!</u> 6!
- **9.** Evaluate. 4! ⋅ 3! ⋅ 2!
- **10.** Solve for *n*, where  $n \in I$ .  $\frac{(n+10)!}{(n+9)!} = 20$
- 11. Solve for *n*, where  $n \in I$ .  $\frac{(n-1)!}{(n-2)!} = 12$
- 12. Evaluate.  ${}_{5}P_{3}$
- **13.** There are twelve different marbles in a bag. Suppose you reach in and draw two marbles one at a time without replacement. How many ways can you draw the two marbles?
- **14.** There are nine different marbles in a bag. Suppose you reach in and draw one at a time, and do this six times. How many ways can you draw the six marbles if you do not replace the marble each time?
- **15.** Evaluate.  $\frac{10!}{3! \cdot 2! \cdot 2! \cdot 2!}$
- **16.** There are 12 members of a student council. How many ways can 5 of the members be chosen to serve on the dance committee?
- **17.** A fun fair requires 6 employees to help move one of the booths. There are 8 people available. How many ways could a group of 6 be chosen?
- **18.** How many ways can you select 2 different flavours of ice-cream for a sundae if there are 16 flavours available?
- **19.** Tad is selecting music for a long car trip. Suppose he has 10 rock albums and 12 hip-hop albums. How many different ways can he select 1 rock album and 2 hip-hop albums?
- **20.** From a standard deck of 52 cards, how many different three-card hands are there with at most one ace?
- **21.** A game has three possible outcomes: *A*, *B*, and *C*. If P(A) = 0.6 and P(B) = 0.2, what is the probability of event *C*?

- **22.** Denis has 15 coins in his pocket, and 6 of them are toonies. He reaches into his pocket and pulls out a coin at random. Determine the probability of the coin being a toonie.
- **23.** Mark plays basketball. He has scored 6 times in 8 shots on basket. He says the odds in favour of him scoring are 3 : 4. Is he right? Explain.
- 24. Ned plays hockey. He has scored 5 times out of 25 shots on goal. He says the odds in favour of him scoring are 1 : 5. Is he right? Explain.
- **25.** The coach of a basketball team claims that, for the next game, the odds in favour of the team winning are 5 : 3, the odds in favour of the team losing are 1 : 3, and the odds against a tie are 7 : 1. Are these odds possible? Explain.
- **26.** From a committee of 12 people, 3 of these people are randomly chosen to be president, vice-president, and secretary. Determine, to the nearest hundredth of a percent, the probability that Pavel, Rashida, and Jerry will be chosen.
- **27.** The probability that Haley will exercise on Sunday is 0.6. The probability that she will go shopping on Sunday is 0.5. The probability that she will do both is 0.3. Determine the probability that Haley will do at least one of these activities on Sunday.
- **28.** Anneliese draws a card from a well-shuffled standard deck of 52 playing cards. Then she draws another card from the deck without replacing the first card. Determine, to the nearest tenth of a percent, the probability that both cards are red.
- **29.** Two cards are drawn without being replaced, from a standard deck of 52 playing cards. Determine, to the nearest hundredth of a percent, the probability of drawing a spade and a diamond.
- 30. Determine the non-permissible value(s) for the rational expression, and then state all restrictions.  $\frac{8x}{2x-x^2}$
- 31. Determine the non-permissible value(s) for the rational expression, and then state all restrictions.  $\frac{5x + 15}{25 - x^2}$
- **32.** Simplify  $\frac{3\nu(1-2\nu)}{7\nu(2-\nu)}$ .
- **33.** Determine the non-permissible values for the rational expression  $\frac{12g^4}{16g^3 4g^2}$ .

**34.** Simplify 
$$\frac{15u^4}{3u^3 - 9u}$$
.

35. Simplify 
$$\frac{21c^2(c-5)}{7c(1+2c)} \cdot \frac{9(2c+1)}{3c}$$
.  
36. Simplify  $\frac{6(5-3d)}{2d+6} + \frac{12d(3d-5)}{d+3}$ .  
37. Simplify  $\frac{15(4j-1)}{10(j+4)} + \frac{3j(1-4j)}{4j+16}$ .  
38. Simplify  $3 + \frac{z}{2z-3}$ .  
39. Simplify  $\frac{3b}{5b+4} + \frac{2}{3}$ .  
40. Simplify  $\frac{4}{3q} + \frac{q-1}{3q+3}$ .  
41. Simplify  $\frac{2}{z^2-1} + \frac{1}{2z(z-1)}$ .  
42. Simplify  $\frac{2m}{m-4} - \frac{8}{m-4}$ .  
43. Simplify  $\frac{4}{y^2-1} - \frac{y}{y+1}$ .  
44. Simplify  $\frac{3n+1}{n^2-1} - \frac{1}{n+1}$ .  
45. Simplify  $\frac{2a}{3-a} - \frac{1}{5(a+2)}$ .

- **46.** Solve the following equation for *x*. State the non-permissible values of the variable.  $\frac{1}{x-2} = \frac{1}{4x+10}$
- 47. Solve the following equation for *x*. State the non-permissible values of the variable.  $\frac{1}{2-x} = \frac{2}{x-20}$
- **48.** Solve the following equation for *x*, if *x* is positive. State the non-permissible values of the variable.

$$\frac{1}{x-2} - \frac{1}{x} = \frac{1}{4}$$

**49.** Solve the following equation for x, if x is negative. State the non-permissible values of the variable.

$$\frac{6}{x^2 - 1} - \frac{1}{x + 1} = \frac{1}{2}$$

## Problem

**1.** A combination lock opens with the correct three-letter code. Each wheel rotates through the letters A to T.

**a**) Suppose each letter can be used only once in a code. How many different codes are possible when repetition is not allowed?

**b**) How many more codes would there be if repetition is allowed?

- 2. Mo has 12 new songs on his mp3 player. How many different 5-song playlists can be created from his new songs, if no songs are repeated? Show your work.
- **3.** A youth hostel has 3 rooms that contain 6, 5, and 4 beds, respectively. How many ways can the 15 players on a hockey team be assigned to these rooms? Show your work.
- **4.** Fifteen camp counselors are signing up for training courses that have only a limited number of spaces. Only 5 people can take the water safety course, 4 people can take the first aid course, 3 people can take the conflict management course, and 3 people can take the astronomy course. How many ways can the 15 counselors be placed in the four courses? Show your work.
- 5. A hockey game has ended in a tie after a 5 min overtime period, so the winner will be decided by a shootout. The coach must decide whether Leanne or Krysta should go first in the shootout. The coach would prefer to use her best scorer first, so she will base her decision on the players' shootout records.

Player	Attempts	<b>Goals Scored</b>
Leanne	12	8
Krysta	14	9

Who should go first? Show your work.

6. A group of students are holding a charity carnival to support a local animal shelter. The students have created a dice game that they call Zing and a card game that they call Bloop. The odds against winning Zing are 3 : 2, and the odds against winning Bloop are 3 : 7. Which game should Lena play? Show your work.

# Math 3201 Answer Section

## **MULTIPLE CHOICE**

1.	ANS:	С
2.	ANS:	А
3.	ANS:	В
4.	ANS:	D
5.	ANS:	В
6.	ANS:	D
7.	ANS:	В
8.	ANS:	А
9.	ANS:	С
10.	ANS:	В
11.	ANS:	В
12.	ANS:	С
13.	ANS:	В
14.	ANS:	В
15.	ANS:	С
16.	ANS:	А
17.	ANS:	С
18.	ANS:	D
19.	ANS:	А
20.	ANS:	D
21.	ANS:	С
22.	ANS:	В
23.	ANS:	С
24.	ANS:	С
25.	ANS:	С
26.	ANS:	D
27.	ANS:	D
28.	ANS:	C
29.	ANS:	В
30.	ANS:	A
31.	ANS:	A
32. 22	ANS:	A
33.	ANS:	В
54. 25	ANS:	D
55. 26	ANS:	A C
30. 25	ANS:	U D
51.	ANS:	D

38.	ANS:	А
39.	ANS:	В
40.	ANS:	В
41.	ANS:	А
42.	ANS:	С
43.	ANS:	D
44.	ANS:	В
45.	ANS:	В
46.	ANS:	D
47.	ANS:	А
48.	ANS:	Α
49.	ANS:	D
50.	ANS:	А
51.	ANS:	D
52.	ANS:	С
53.	ANS:	А
54.	ANS:	А
55.	ANS:	D
56.	ANS:	D
57.	ANS:	А
58.	ANS:	D
59.	ANS:	С
60.	ANS:	С
61.	ANS:	А
62.	ANS:	D
63.	ANS:	С
64.	ANS:	В
65.	ANS:	С
66.	ANS:	С
67.	ANS:	D
68.	ANS:	А
69.	ANS:	В
70.	ANS:	D
71.	ANS:	D
72.	ANS:	В
73.	ANS:	А
74.	ANS:	С
75.	ANS:	В
76.	ANS:	D
77.	ANS:	В
78.	ANS:	В
79.	ANS	D

80.	ANS:	D
81.	ANS:	А
82.	ANS:	С
83.	ANS:	D
84.	ANS:	С
85.	ANS:	D
86.	ANS:	С
87.	ANS:	С
88.	ANS:	А
89.	ANS:	А

#### SHORT ANSWER

**1.** ANS:  
$$A = \{x \mid 0 < x < 22, x \in \mathbb{R}\}$$

- 2. ANS:  $A = \{y \mid y = 2x, -10 \le x \le -2, x \in I\}$
- **3.** ANS: Fifteen students like only mathematics and 12 students like only science.
- **4.** ANS:

There are 11 students who plan to ride both the roller coaster and the vertical drop.

- **5.** ANS:  $n(B \setminus S \setminus C) = 22\%$
- **6.** ANS:

785 people use public transit only.



- **7.** ANS: 360
- 8. ANS:

	$\frac{1}{6}$
9.	ANS: 288
10.	ANS: 10
11.	ANS: 13
12.	ANS: 60
13.	ANS: 132
14.	ANS: 60 480
15.	ANS: 75 600
16.	ANS: 792
17.	ANS: 28
18.	ANS: 120
19.	ANS: 660
20.	ANS: 21 808
21.	ANS: $P(C) = 0.2$

**22.** ANS: 0.4

#### **23.** ANS:

No. If Mark had scored 6 times out of 8, then he had not scored 2 times out of 8. Therefore, the odds in favour are 6:2 or 3:1.

### **24.** ANS:

No. If Ned had scored 5 times out of 25, then he had not scored 20 times out of 25. Therefore, the odds in favour are 5:20 or 1:4.

### **25.** ANS:

Yes. The probability of a win is 5 in 8 (62.5%), the probability of a loss is 1 in 4 (25%), and the probability of a tie is 1 in 8 (12.5%). The probabilities add up to 100%.

**26.** ANS:

0.45%

- 27. ANS: 0.8
  - 0.0

**28.** ANS: 24.5%

- **29.** ANS: 6.37%
- **30.** ANS:  $x \neq 2, x \neq 0$
- **31.** ANS:  $x \neq -5, x \neq 5$
- 32. ANS:  $\frac{3-6v}{14-7v}, v \neq 0, 2$
- **33.** ANS:

 $g \neq 0, \frac{1}{4}$ 

- 34. ANS:  $\frac{5u^3}{u^2 - 3}, u \neq 0, \pm \sqrt{3}$
- **35.** ANS:  $9(c-5), c \neq 0, -\frac{1}{2}$

**36.** ANS: 
$$\frac{1}{-4d}, d \neq 0, -3, \frac{5}{3}$$

**37.** ANS: 
$$\frac{2}{-j}, j \neq 0, -4, \frac{1}{4}$$

**38.** ANS: 
$$\frac{7z-9}{2z-3}, z \neq \frac{3}{2}$$

**39.** ANS: 
$$\frac{19b+8}{15b+12}, b \neq -\frac{4}{5}$$

**40.** ANS:  
$$\frac{q^2 + 3q + 4}{3q(q+1)}, q \neq 0, -1$$

- **41.** ANS:  $\frac{5z+1}{2z(z^2-1)}, z \neq 0, -1, 1$
- **42.** ANS: 2,  $m \neq 4$
- **43.** ANS:  $\frac{4-y^2+y}{2}$ ,

$$\frac{4 - y^2 + y}{y^2 - 1}, y \neq -1, 1$$

- **44.** ANS:  $\frac{2}{n-1}, n \neq -1, 1$
- **45.** ANS:  $\frac{10a^2 + 21a - 3}{5(a+2)(3-a)}, a \neq -2, 3$
- **46.** ANS:

$$x = -4; x \neq -\frac{5}{2}, x \neq 2$$

- **47.** ANS:  $x = 8; x \neq 2, x \neq 20$
- **48.** ANS:  $x = 4; x \neq 0, x \neq 2$
- **49.** ANS:  $x = -5; x \neq 1, x \neq -1$

#### PROBLEM

**1.** ANS:

a) There are 20 letters from A to T. The number of different codes, *C*, is related to the number of letters from which to select on each wheel of the lock, *W*:  $C = W_1 \cdot W_2 \cdot W_3$  $C = 20 \cdot 19 \cdot 18$ C = 6840There are 6840 different three-letter codes on this type of lock.

**b**) The number of different codes, *R*, is related to the number of letters from which to select on each wheel of the lock, *X*:

 $R = X_1 \cdot X_2 \cdot X_3$   $R = 20 \cdot 20 \cdot 20$  R = 8000 R - C = 8000 - 6840 R - C = 1160There are 1160 more codes if repetition is allowed.

## **2.** ANS:

There are 12 songs and 5 positions they can be placed in. Let *A* represent the number of arrangements:

$$A = {}_{12}P_{5}$$

$$A = \frac{12!}{7!}$$

$$A = 12 \cdot 11 \cdot 10 \cdot 9 \cdot$$

$$A = 95040$$

8

There are 95 040 different 5-song playlists that can be created from 12 songs

## **3.** ANS:

For the last room, there are 15 players and 4 beds. Order does not matter.

$${}_{15}C_4 = \frac{15!}{4! \cdot 11!}$$

$${}_{15}C_4 = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 11!}$$

$${}_{15}C_4 = \frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2}$$

$${}_{15}C_4 = 15 \cdot 7 \cdot 13$$

$${}_{15}C_4 = 1365$$

For the first room, there are now 11 players and 6 beds. Order does not matter.

$$\begin{split} _{11}C_6 &= \frac{11!}{5!\cdot 6!} \\ _{11}C_6 &= \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6!}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 6!} \\ _{11}C_6 &= \frac{11\cdot 10\cdot 9\cdot 8\cdot 7}{5\cdot 4\cdot 3\cdot 2} \\ _{11}C_6 &= 11\cdot 3\cdot 2\cdot 7 \\ _{11}C_6 &= 462 \end{split}$$

The remaining 5 players share the middle room.

Using the Fundamental Counting Principle, the product of  ${}_{15}C_4$  and  ${}_{11}C_6$  is the number of ways the players can be assigned to the three rooms.

$$_{15}C_4 \cdot _{11}C_6 = 1365 \cdot 462$$
  
 $_{15}C_4 \cdot _{11}C_6 = 630\,630$ 

There are 630 630 ways to assign the 15 players to these rooms.

**4.** ANS:

5 of 15 people in water safety:  $\begin{pmatrix} 15\\5 \end{pmatrix}$ 

4 of the remaining 10 people in first aid:  $\begin{pmatrix} 10 \\ 4 \end{pmatrix}$ 

3 of the remaining 6 people in conflict management:  $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ 

3 people in astronomy:  $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ 

Let C represent the number of ways to place the 15 counselors in the four courses:

$$C = \begin{pmatrix} 15\\5 \end{pmatrix} \begin{pmatrix} 10\\4 \end{pmatrix} \begin{pmatrix} 6\\3 \end{pmatrix} \begin{pmatrix} 3\\3 \end{pmatrix}$$
$$C = 3003 \cdot 210 \cdot 20 \cdot 1$$
$$C = 12612600$$

There are 12 612 600 ways to place the counselors in the four courses.

## 5. ANS:

Leanne has 12 attempts and has scored 8 goals. This means that she has 12 - 8 or 4 attempts where she did not score. The odds in favour of her scoring are 2 : 1.

Krysta has 14 attempts and has scored 9 goals. This means that she has 14 - 9 or 5 attempts where she did not score. The odds in favour of her scoring are 9:5.

The probability that Leanne will score is  $\frac{2}{3}$  or about 0.667.

The probability that Krysta will score is  $\frac{9}{14}$  or about 0.643.

Since 0.667 > 0.643, there is a better chance that Leanne will score. Therefore, Leanne should go first.

**6.** ANS:

The odds against winning Zing are 3:2. The total number of outcomes is 3+2 or 5. So, if she plays Zing 5 times, she is likely to lose 3 times and win 2 times.

The probability of winning Zing is  $\frac{2}{5}$  or 0.4.

The odds against winning Bloop are 3:7. The total number of outcomes is 3+7 or 10. So, if she plays Bloop 10 times, she is likely to lose 3 times and win 7 times.

The probability of winning Bloop is  $\frac{7}{10}$  or 0.7.

Lena should play Bloop, since she is more likely to win.