Math 3208

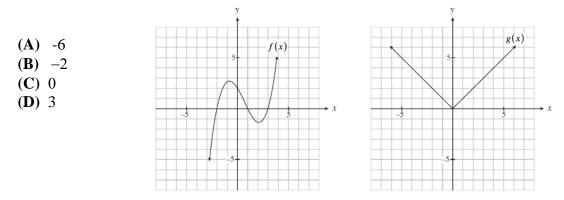
Midtem Review

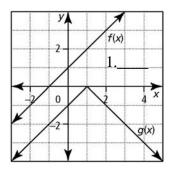
Pre-Calculus

- **1.** From the graph, what is the value of (f g)(2)?
 - (**A**) −3 (**B**) 0
 - $(\mathbf{D}) \mathbf{0}$
 - (C) 2
 - **(D)** 4

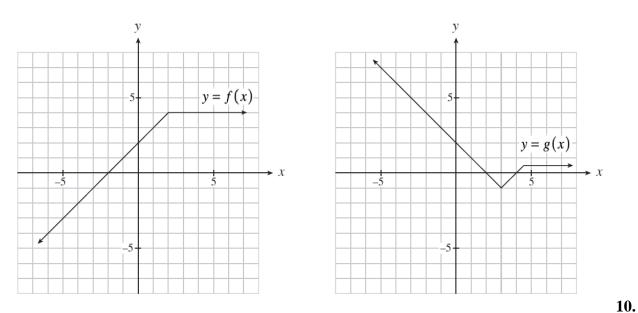
2. Given $f(x) = x^2 + 2$ and g(x) = x - 5, which equation represents h(x) = (f + g)(x)?

- (A) $h(x) = 2x^2 5$ (B) $h(x) = x^2 + x - 3$ (C) $h(x) = x^2 + x - 5$ (D) $h(x) = x^2 + 2x - 5$
- 3. Let f(x) = x 1 and $g(x) = x^2 1$. Determine the non-permissible values of $y = \left(\frac{f}{g}\right)(x)$. (A) 1 (B) -1 (C) ± 1 (D) none
- If $f(x) = \sqrt{3x-1}$ and $g(x) = x^2$, which is the domain of $m(x) = \frac{g(x)}{f(x)}$? (A) $\{x \mid x > 0, x \in \mathbb{R}\}$ (B) $\{x \mid x \neq 0, x \in \mathbb{R}\}$ (C) $\{x \mid x > \frac{1}{3}, x \in \mathbb{R}\}$ (D) $\{x \mid x \neq \frac{1}{3}, x \in \mathbb{R}\}$
- 5. Given f(x) = x + 2 and $g(x) = x^2 + 3x 1$, determine the value of f(g(3)).
 - (A) 16 (B) 17 (C) 19 (D) 39
- **6**. The graphs of f(x) and g(x) are given below. Determine f(g(-3)).





- 7. Given the functions f(x) = 4x 6 and $g(x) = (x 2)^2$, determine h(x) = f(x) g(x).
- 8. Given the function $f(x) = 2x^2 3x + 7$, determine f(x+h) f(x).
- 9. Use the graphs of y = f(x) and y = g(x) shown to sketch the graph of y = f(x)g(x). Clearly indicate a sufficient number of points to get an accurate representation of your graph.



Consider the functions $f(x) = x^2 + 6x + 8$ and g(x) = x + 4.

- (i) Determine the equation of the function $h(x) = \frac{f(x)}{g(x)}$.
- (ii) State the domain and range of h(x).

11. Given f(x) = x + 1, g(x) = x - 5, and h(x) = x - 4, determine each combined function.

(i) y = f(x)g(x)h(x) (ii) $y = \frac{f(x)g(x)}{h(x)}$

12. Given f(-1) = 7, f(7) = 5, f(3) = 0, g(-1) = 3, g(7) = -1, and g(5) = -2, find each value.

a)
$$f(g(7))$$
 b) $f(g(-1))$

14. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, $h(x) = \frac{1}{x}$, determine a) g(f(x)) and state the domain b) h(f(x)) and state the domain

15. Find two functions, f(x) and g(x), such that $f(g(x)) = (2x + 3)^2 - 5$.

Limits & Continuity

16. Determine the *x*-values (if any) at which f is not continuous. Which of the discontinuities are removable?

a.
$$f(x) = \frac{x}{x^2 - x}$$
 b. $f(x) = \begin{cases} 2x + 1, & x \ge -2\\ \frac{2x}{x + 2}, & x < -2 \end{cases}$

17. For what value(s) of "k" is the following function continuous everywhere?

$$f(x) = \begin{cases} kx^2 + 1, & x > 2\\ -x, & x \le 2 \end{cases}$$

18. Find the values of *a* and *b* such that f(x) is continuous.

$$f(x) = \begin{cases} ax+1, & x < 2\\ bx-1, & x = 2\\ ax+b-6, & x > 2 \end{cases}$$

19. Use the definition of continuity to determine if f(x) is continuous at x = 0. Find any other points of discontinuity and state the type.

$$f(x) = \begin{cases} \frac{x-3}{x^2-1}, & x < 0\\ 2x+1, & x = 0\\ (x+2)^2 - 1, & x > 0 \end{cases}$$

20. Use the definition of continuity to determine if f(x) is continuous at x = 1

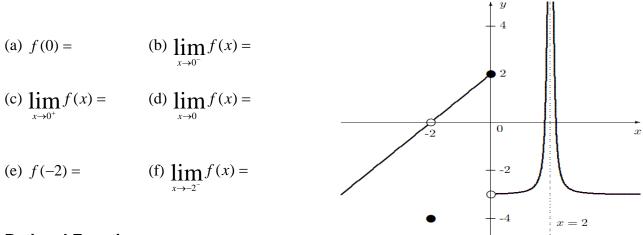
$$f(x) = \begin{cases} x^2 + 1, & x < 1 \\ 2x, & x = 1 \\ 3x^2 - 1, & x > 1 \end{cases}$$

21. Evaluate the following limits, showing the workings. Assign ∞ or $-\infty$ as appropriate.

(A)
$$\lim_{x \to -\infty} \frac{3x+1}{\sqrt{4x^2-5}}$$
 (B) $\lim_{x \to -1^-} \frac{2x^2+x-1}{x^2+2x+1}$ (C) $\lim_{x \to -2} \frac{3-\sqrt{x+7}}{x-2}$

(D)
$$\lim_{x \to 1} \frac{2x + \sqrt{x^2 + 3}}{\sqrt{x^3 + 1}}$$
 (E) $\lim_{x \to -3^-} \frac{x + 3}{|x + 3|}$

22. Use the graph of y = f(x) below to determine each of the following. Label the limits as ∞ or $-\infty$ where appropriate. If the limit does not exist or the value of the function is undefined, indicate this.



Rational Functions

23. For each function below

a. Find the horizontal and vertical asymptotes of the function.

b. Determine x and y intercepts. c. Determine any POD.

d. Sketch

A)
$$f(x) = \frac{2x-2}{x^2-1}$$
 B) $f(x) = \frac{x^2-9}{x^2+2x-3}$

The Derivative

24. For each function given, determine whether f(x) is continuous and whether f(x) is differentiable at the given x value.

(a)
$$f(x) = \begin{cases} x^2 + 2 & \text{if } x \le -1 \\ -2x + 1 & \text{if } x > -1 \end{cases}$$
 (b) $f(x) = \begin{cases} \frac{4x^2 - 8x + 4}{x^2 - 1} & \text{if } x < 1 \\ 3x^2 - 4x + 1 & \text{if } x \ge 1 \end{cases}$

25. Using the Limit Definition of the derivative, differentiate the following functions.

(a)
$$f(x) = 2x^2 + 3x + 5$$
 (b) $f(x) = \sqrt{3 + 2x}$ (c) $f(x) = \frac{1}{x}$

- If an arrow is shot upward on the moon with a velocity of 58 m/s, its height (in meters) after *t* seconds is given by $h(t) = 58t 0.83t^2$.
 - (a) What is the instantaneous velocity at t = 1 s?
 - (b) What is the instantaneous velocity at t = 3s?
 - (c) What is the average velocity from t = 1 s to t = 3 s?
- 27. (a) Determine the equation of the tangent line to the curve $y = x^3 2x + 1$ at the point (-1, 2).
 - (b) Determine the equation of the normal line (to the same curve at the same point). [2]
- 28. The position of a particle is given by the function $s(t) = t^3 9t^2 + 15t$ where t is measured in seconds and s is measured in feet.
 - (a) Determine the velocity at time *t*. (b) What is the velocity at t = 3 seconds?
 - (c) When is the particle at rest? (d) When is the particle moving in a positive direction?
 - (e) Determine the acceleration at time *t*. (f) When is the particle speeding up?
- 29. Differentiate the following.
 - (a) $y = (x^2 4x)(3x^2 + 5x + 2)$ (b) $y = 3\sqrt{x}(2 x^2)$ c. $y = \sqrt{3x 1}$
- 30. Differentiate the following.

(a)
$$2x + 3y^2 = 1$$
 (b) $x^3 - 5xy^2 = 2y$ c. $y = \frac{x^2 + 3}{\sqrt{x}}$

Applications of the Derivative

- 31. Given the function $f(x) = x^3 + 2x^2 4x + 1$
 - Determine location of local max/min points
 - Determine intervals of increase/decrease
 - Determine location of inflection ppoints
 - Determine intervals of concave up/down