Midtem Review

## Pre-Calculus

1. From the graph, what is the value of $(f-g)(2)$ ?
(A) -3
(B) 0
(C) 2
(D) 4

2. Given $f(x)=x^{2}+2$ and $g(x)=x-5$, which equation represents $h(x)=(f+g)(x)$ ?
(A) $h(x)=2 x^{2}-5$
(B) $h(x)=x^{2}+x-3$
(C) $h(x)=x^{2}+x-5$
(D) $h(x)=x^{2}+2 x-5$
3. Let $f(x)=x-1$ and $g(x)=x^{2}-1$. Determine the non-permissible values of $y=\left(\frac{f}{g}\right)(x)$.
(A) 1
(B) -1
(C) $\pm 1$
(D) none

- If $f(x)=\sqrt{3 x-1}$ and $g(x)=x^{2}$, which is the domain of $m(x)=\frac{g(x)}{f(x)}$ ?
(A) $\{x \mid x>0, x \in \mathrm{R}\}$
(B) $\quad\{x \mid x \neq 0, x \in \mathrm{R}\}$
(C) $\left\{x \left\lvert\, x>\frac{1}{3}\right., x \in \mathrm{R}\right\}$
(D) $\left\{x \left\lvert\, x \neq \frac{1}{3}\right., x \in \mathrm{R}\right\}$

5. Given $f(x)=x+2$ and $g(x)=x^{2}+3 x-1$, determine the value of $f(g(3))$.
(A) 16
(B) 17
(C) 19
(D) 39
6. The graphs of $f(x)$ and $g(x)$ are given below. Determine $f(g(-3))$.
(A) -6
(B) -2
(C) 0
(D) 3


7. Given the functions $f(x)=4 x-6$ and $g(x)=(x-2)^{2}$, determine $h(x)=f(x)-g(x)$.
8. Given the function $f(x)=2 x^{2}-3 x+7$, determine $f(x+h)-f(x)$.
9. Use the graphs of $y=f(x)$ and $y=g(x)$ shown to sketch the graph of $y=f(x) g(x)$. Clearly indicate a sufficient number of points to get an accurate representation of your graph.


10. 

Consider the functions $f(x)=x^{2}+6 x+8$ and $g(x)=x+4$.
(i) Determine the equation of the function $h(x)=\frac{f(x)}{g(x)}$.
(ii) State the domain and range of $h(x)$.
11. Given $f(x)=x+1, g(x)=x-5$, and $h(x)=x-4$, determine each combined function.
(i) $y=f(x) g(x) h(x)$
(ii) $y=\frac{f(x) g(x)}{h(x)}$
12. Given $f(-1)=7, f(7)=5, f(3)=0, g(-1)=3, g(7)=-1$, and $g(5)=-2$, find each value.
a) $f(g(7))$
b) $\quad f(g(-1))$
14. Given $f(x)=\sqrt{x}$ and $g(x)=x^{2}-1, h(x)=\frac{1}{x}$, determine
a) $g(f(x))$ and state the domain
b) $h(f(x))$ and state the domain
15. Find two functions, $f(x)$ and $g(x)$, such that $f(g(x))=(2 x+3)^{2}-5$.

## Limits \& Continuity

16. Determine the $x$-values (if any) at which $f$ is not continuous. Which of the discontinuities are removable?
a. $f(x)=\frac{x}{x^{2}-x}$
b. $\quad f(x)= \begin{cases}2 x+1, & x \geq-2 \\ \frac{2 x}{x+2}, & x<-2\end{cases}$
17. For what value(s) of " k " is the following function continuous everywhere?

$$
f(x)= \begin{cases}k x^{2}+1, & x>2 \\ -x, & x \leq 2\end{cases}
$$

18. Find the values of $a$ and $b$ such that $f(x)$ is continuous.

$$
f(x)= \begin{cases}a x+1, & x<2 \\ b x-1, & x=2 \\ a x+b-6, & x>2\end{cases}
$$

19. Use the definition of continuity to determine if $f(x)$ is continuous at $x=0$. Find any other points of discontinuity and state the type.

$$
f(x)= \begin{cases}\frac{x-3}{x^{2}-1}, & x<0 \\ 2 x+1, & x=0 \\ (x+2)^{2}-1, & x>0\end{cases}
$$

20. Use the definition of continuity to determine if $f(x)$ is continuous at $x=1$

$$
f(x)= \begin{cases}x^{2}+1, & x<1 \\ 2 x, & x=1 \\ 3 x^{2}-1, & x>1\end{cases}
$$

21. Evaluate the following limits, showing the workings. Assign $\infty$ or $-\infty$ as appropriate.
(A) $\lim _{x \rightarrow-\infty} \frac{3 x+1}{\sqrt{4 x^{2}-5}}$
(B) $\lim _{x \rightarrow-1^{-}} \frac{2 x^{2}+x-1}{x^{2}+2 x+1}$
(C) $\lim _{x \rightarrow-2} \frac{3-\sqrt{x+7}}{x-2}$
(D) $\lim _{x \rightarrow 1} \frac{2 x+\sqrt{x^{2}+3}}{\sqrt{x^{3}+1}}$
(E) $\quad \lim _{x \rightarrow-3^{-}} \frac{x+3}{|x+3|}$
22. Use the graph of $y=f(x)$ below to determine each of the following. Label the limits as $\infty$ or $-\infty$ where appropriate. If the limit does not exist or the value of the function is undefined, indicate this.
(a) $f(0)=$
(b) $\lim _{x \rightarrow 0^{-}} f(x)=$
(c) $\lim _{x \rightarrow 0^{+}} f(x)=$
(d) $\lim _{x \rightarrow 0} f(x)=$
(e) $f(-2)=$
(f) $\lim _{x \rightarrow-2^{-}} f(x)=$

## Rational Functions


23. For each function below
a. Find the horizontal and vertical asymptotes of the function.
b. Determine x and y intercepts.
c. Determine any POD.
d. Sketch
A) $\quad f(x)=\frac{2 x-2}{x^{2}-1}$
B) $\quad f(x)=\frac{x^{2}-9}{x^{2}+2 x-3}$

## The Derivative

24. For each function given, determine whether $f(x)$ is continuous and whether $f(x)$ is differentiable at the given $x$ value.
(a) $\quad f(x)= \begin{cases}x^{2}+2 & \text { if } x \leq-1 \\ -2 x+1 & \text { if } x>-1\end{cases}$
(b) $\quad f(x)= \begin{cases}\frac{4 x^{2}-8 x+4}{x^{2}-1} & \text { if } x<1 \\ 3 x^{2}-4 x+1 & \text { if } x \geq 1\end{cases}$
25. Using the Limit Definition of the derivative, differentiate the following functions.
(a) $f(x)=2 x^{2}+3 x+5$
(b) $f(x)=\sqrt{3+2 x}$
(c) $f(x)=\frac{1}{x}$

26 If an arrow is shot upward on the moon with a velocity of $58 \mathrm{~m} / \mathrm{s}$, its height (in meters) after $t$ seconds is given by $h(t)=58 t-0.83 t^{2}$.
(a) What is the instantaneous velocity at $t=1 \mathrm{~s}$ ?
(b) What is the instantaneous velocity at $t=3 \mathrm{~s}$ ?
(c) What is the average velocity from $t=1 \mathrm{~s}$ to $t=3 \mathrm{~s}$ ?
27. (a) Determine the equation of the tangent line to the curve $y=x^{3}-2 x+1$ at the point ( $-1,2$ ).
(b) Determine the equation of the normal line (to the same curve at the same point).
[2]
28. The position of a particle is given by the function $s(t)=t^{3}-9 t^{2}+15 t$ where $t$ is measured in seconds and $s$ is measured in feet.
(a) Determine the velocity at time $t$.
(b) What is the velocity at $t=3$ seconds?
(c) When is the particle at rest?
(d) When is the particle moving in a positive direction?
(e) Determine the acceleration at time $t$. (f) When is the particle speeding up?
29. Differentiate the following.
(a) $y=\left(x^{2}-4 x\right)\left(3 x^{2}+5 x+2\right)$
(b) $y=3 \sqrt{x}\left(2-x^{2}\right)$
c. $y=\sqrt{3 x-1}$
30. Differentiate the following.
(a) $2 x+3 y^{2}=1$
(b) $x^{3}-5 x y^{2}=2 y$
c. $y=\frac{x^{2}+3}{\sqrt{x}}$

## Applications of the Derivative

31. Given the function $f(x)=x^{3}+2 x^{2}-4 x+1$

- Determine location of local max/min points
- Determine intervals of increase/decrease
- Determine location of inflection ppoints
- Determine intervals of concave up/down

