Multiple Choice Answers:

1. D
2. B
3. B
4. C
5. C
6. C

7. $f(x) = 4x-6$ and $g(x) = (x-2)^2$

$h(x) = f(x) - g(x)$

$= 4x-6-(x-2)^2$

$= 4x-6-[x^2-4x+4]$

$= 4x-6+4x-x^2-4$

$= -x^2+8x-10$ ■

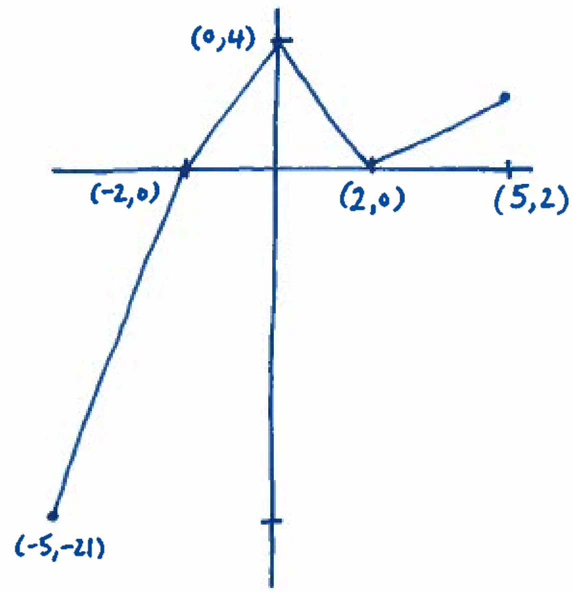
8. $f(x) = 2x^2-3x+7$

$f(x+h) = 2(x+h)^2-3(x+h)+7$
$= 2[x^2+2xh+h^2]-3x-3h+7$
$= 2x^2+4xh+2h^2-3x-3h+7$
$= 2x^2+2h^2+4xh-3x-3h+7$

$f(x+h)-f(x) = \cancel{2x^2}+2h^2+4xh-\cancel{3x}-3h+\cancel{7}-\cancel{2x^2}+\cancel{3x}-\cancel{7}$
$= 2h^2+4xh-3h$
$= h(2h+4x-3)$

9.

$f(x)$	$g(x)$	$f(x)\cdot g(x)$
$(0,2)$	$(0,2)$	$(0,4)$
$(2,4)$	$(2,0)$	$(2,0)$
$(-2,0)$	$(-2,4)$	$(-2,0)$
$(5,4)$	$(5,\frac{1}{2})$	$(5,2)$
$(-5,-3)$	$(-5,7)$	$(-5,-21)$



10. $f(x) = x^2+6x+8$ and $g(x) = x+4$.

(i) $h(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2+6x+8}{x+4} = \dfrac{\cancel{(x+4)}(x+2)}{\cancel{x+4}}$

$= x+2$ ■

(ii) Domain: $x \neq -4$

Range: $h(x) = x+2$ $\Rightarrow$ $y \neq -2$
$h(-4) = -4+2 = -2$

11. $f(x) = x+1$, $g(x) = x-5$, and $h(x) = x-4$.

(i) $y = f(x)\,g(x)\,h(x)$

$= (x+1)(x-5)(x-4)$

$= (x^2-4x-5)(x-4)$

$= x^3-4x^2-4x^2+16x-5x+20$

$= x^3-8x^2+11x+20$ ■

(ii) $y = \dfrac{f(x)\,g(x)}{h(x)}$

$= \dfrac{(x+1)(x-5)}{(x-4)}$

$= \dfrac{x^2-4x-5}{x-4}$

12. $f(-1) = 7$, $f(7) = 5$, $f(3) = 0$

$g(-1) = 3$, $g(7) = -1$, $g(5) = -2$.

(a) $f(g(7))$

$f(g(7)) = f(-1) = 7$

(b) $f(g(-1))$

$f(g(-1)) = f(3) = 0$

14. $f(x) = \sqrt{x}$, $g(x) = x^2 - 1$, $h(x) = \dfrac{1}{x}$

(a) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1$

Domain: $x \geq 0$

(b) $h(f(x)) = h(\sqrt{x}) = \dfrac{1}{\sqrt{x}}$

Domain: $x > 0$

15. $f(g(x)) = (2x+3)^2 - 5$

Then we know $g(x) = 2x+3$

and $f(x) = x^2 - 5$ ■

16.

(a) $x^2 - x = 0$

$x(x-1) = 0$

$x = 0 \quad \Big\{ \quad x = 1$

$$\lim_{x \to 0^+} \frac{x}{x^2 - x} = \frac{1}{x-1} = -1$$

$$\lim_{x \to 0^-} \frac{x}{x^2 - x} = \frac{1}{x-1} = -1$$

$f(0) =$ undefined

$\therefore$ Removable discontinuity.

(b) $$\lim_{x \to 1^+} \frac{1}{x-1} = +\infty$$

$$\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$$

$\therefore$ Non-removable discontinuity.

(b) $\lim_{x \to -2^-} \frac{2x}{x+2} = +\infty$

$\lim_{x \to -2^+} 2x+1 = 2(-2)+1 = -3$

$\therefore$ Non-removable (infinite)

17. $\lim_{x \to 2^-} -x = -2$

$\lim_{x \to 2^+} kx^2+1 = 4k+1$

$-2 = 4k+1$
$-2-1 = 4k$
$\frac{-3}{4} = \frac{4k}{4}$

$\rightarrow k = -3/4$

18. $\lim_{x \to 2^-} ax+1 = 2a+1$

$\lim_{x \to 2^+} ax+b-6 = 2a+b-6$

$f(2) = 2b-1$

$\cancel{2a}+1 = \cancel{2a}+b-6$
$1+6 = b$
$\boxed{7 = b}$

$$2a + b - 6 = 2b - 1$$

$$2a + 7 - 6 = 2(7) - 1$$

$$2a + 1 = 14 - 1$$

$$2a + 1 = 13$$

$$\frac{2a}{2} = \frac{12}{2}$$

$$\boxed{a = 6}$$

$\therefore$ $a = 6$ and $b = 7$

19. f is continuous at $x=a$ if $\lim_{x \to a} f(x) = f(a)$.

$f(0) = 2(0) + 1 = 1$, so defined at $x=0$.

$$\lim_{x \to 0^-} \frac{x-3}{x^2-1} = \frac{-3}{-1} = 3$$

$$\lim_{x \to 0^+} (x+2)^2 - 1 = 4 - 1 = 3$$

$\therefore$ Since $3 \neq 1$, it is not continuous and we have a removable discontinuity at $x=0$.

We also need to check for other points of discontinuity:

$$f(x) = \frac{x-3}{x^2-1} = \frac{x-3}{(x+1)(x-1)}$$

$x+1=0$ | $x-1=0$
$x=-1$ | $x=1$

$\therefore$ Both $x=-1$ and $x=1$ are non-removable discontinuities.

We now check the conditions:

$f(-1) = 2(-1) + 1 = -1$

$$\lim_{x \to -1^-} \frac{x-3}{x^2-1} = \frac{-4}{0} = \infty$$

$$\lim_{x \to -1^+} (x+2)^2 - 1 = 0$$

20. $f(1) = 2(1) = 2$

$$\lim_{x \to 1^-} x^2+1 = (1)^2+1 = 2$$

$$\lim_{x \to 1^+} 3x^2-1 = 3-1 = 2$$

Since $\lim_{x \to 1} f(x) = f(x)$, then our

function is continuous at $x=1$ ∎

21.

(a) $$\lim_{x \to -\infty} \frac{3x+1}{\sqrt{4x^2-5}} = \lim_{x \to -\infty} \frac{3x/_{-x} + 1/_{-x}}{\sqrt{4x^2/x^2 - 5/x^2}} = \frac{-3+0}{2-0} = \frac{-3}{2}$$ ∎

(b) $$\lim_{x \to -1^-} \frac{2x^2+x-1}{x^2+2x+1} = \lim_{x \to -1^-} \frac{(2x-1)\cancel{(x+1)}}{(x+1)\cancel{(x+1)}} = \lim_{x \to -1^-} \frac{2x-1}{x+1} = -\infty$$ ∎

$$\left(\frac{k}{0} \Rightarrow +\infty \text{ or } -\infty\right)$$

(c) $$\lim_{x \to +2} \frac{3-\sqrt{x+7}}{x-2} = \lim_{x \to +2} \frac{3-\sqrt{x+7}}{x-2} \cdot \frac{(3+\sqrt{x+7})}{(3+\sqrt{x+7})}$$

$$= \lim_{x \to 2} \frac{9-(x+7)}{(x-2)(3+\sqrt{5})} = \lim_{x \to 2} \frac{2-x}{(x-2)(3+\sqrt{5})}$$

$$\boxed{\underline{\text{Note:}}\quad 2-x = -(x-2)}$$

$$= \lim_{x \to 2} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(3+\sqrt{5})} = \frac{-1}{3+\sqrt{5}}$$ ∎

(d) $\lim_{x \to 1} \dfrac{2x + \sqrt{x^2+3}}{\sqrt{x^3+1}} = \dfrac{2(1) + \sqrt{1+3}}{\sqrt{1+1}}$

$= \dfrac{2+2}{\sqrt{2}} = \dfrac{4}{\sqrt{2}}$ ■

(e) $\lim_{x \to -3^-} \dfrac{x+3}{|x+3|} = \lim_{x \to -3^-} \dfrac{\cancel{x+3}}{-\cancel{(x+3)}} = -1$ ■

22.
(a) $f(0) = 2$

(b) $\lim_{x \to 0^-} f(x) = 2$

(c) $\lim_{x \to 0^+} f(x) = -3$

(d) $\lim_{x \to 0} f(x) = \text{DNE}$

(e) $f(-2) = -4$

(f) $\lim_{x \to -2^-} f(x) = 0$ ■

23. $f(x) = \dfrac{2x-2}{x^2-1}$ Since the degree of the numerator is less than the degree of the denominator, then $y=0$ is a H.A.
(A)

$x^2 - 1 = 0 \Rightarrow (x+1)(x-1) = 0$, then
$x = -1$ is a V.A.

<u>y-int:</u> $x = 0$ then $y = \frac{2(0)-2}{(0)^2-1} = \frac{-2}{-1} = 2.$

<u>x-int:</u> $2x - 2 = 0$ then $2(x-1) = 0$

There is no x-int.

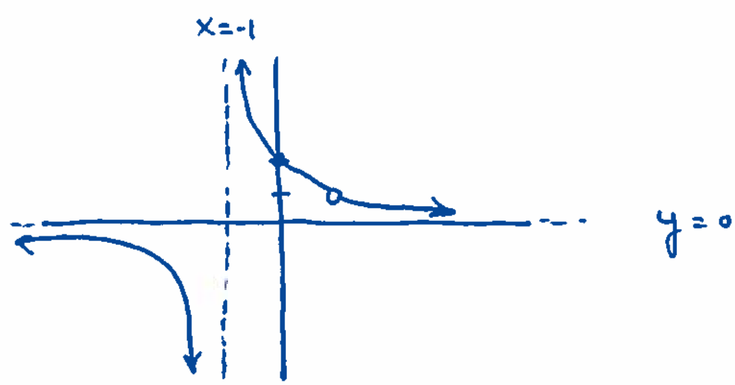
$$y = \frac{2x-2}{x^2-1} = \frac{2\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{2}{(x+1)}$$

There is a P.O.D at $x = 1$, $y = 1 \Rightarrow (1, 1)$.

$$\lim_{x \to -1^+} \frac{2}{x+1} = +\infty$$

$$\lim_{x \to -1^-} \frac{2}{x+1} = -\infty$$

<u>Sketch:</u>



(B) $f(x) = \dfrac{x^2-9}{x^2+2x-3}$

Since the degree of numerator and denominator are equal, then a H.A. occurs at:

$$y = \frac{1}{1} = 1.$$

$x^2+2x-3=0$ V.A occur at

$(x+3)(x-1)=0$ $x=1$.

y-int: $y = \dfrac{(0)^2-9}{(0)^2+2(0)-3} = \dfrac{-9}{-3} = 3.$

X-int: $x^2-9=0 \Rightarrow (x+3)(x-3)=0$

Then, $x=3$ is a x-int.

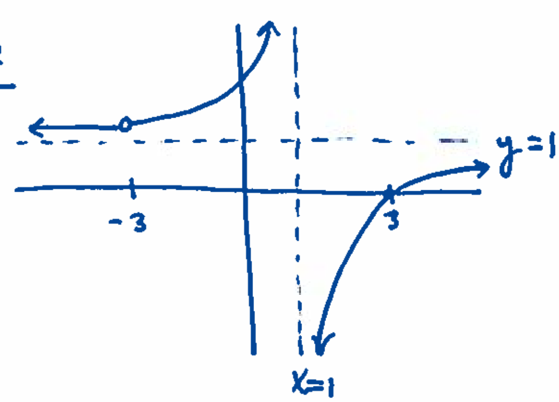
$f(x) = \dfrac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x-1)} = \dfrac{x-3}{x-1}$. So, there

is a P.O.D. at $x=-3$, $y=3/2$ then $(-3, 3/2)$

$\lim\limits_{x\to 1^+} \dfrac{x-3}{x-1} = -\infty$

$\lim\limits_{x\to 1^-} \dfrac{x-3}{x-1} = +\infty$

Sketch:



24. (a) $\lim_{x \to -1^-} x^2+2 = (-1)^2+2 = 3$

$\lim_{x \to -1^+} -2x+1 = -2(-1)+1 = 2+1 = 3$

$f(-1) = (-1)^2 + 2 = 3$

$\therefore$ $f(x)$ is continuous at $x = -1$

(b) $\lim_{x \to 1^-} \frac{4x^2-8x+4}{x^2-1} = \lim_{x \to 1^-} \frac{4(x-1)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = 0$

$\lim_{x \to 1^+} 3x^2-4x+1 = \lim_{x \to 1^+} 3(1)^2-4(1)+1 = 0$

$f(x) = f(1) = 3(1)^2-4(1)+1 = 0$

$\therefore$ $f(x)$ is continuous at $x = 1$

Remember to check $x=-1$ since $(x+1)(x-1)=0$, so we use both values.

$\lim_{x \to -1^-} \frac{4x^2-8x+4}{x^2-1} = \frac{4(-1)^2-8(-1)+4}{(-1)^2-1} = +\infty$

$\lim_{x \to -1^+} 3x^2-4x+1 = 3(-1)^2-4(-1)+1 = 3+4+1 = 8$

$\therefore$ $f(x)$ is not continuous at $x = -1$ ■

25.

(a) $f(x) = 2x^2 + 3x + 5$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 + 3(x+h) + 5 - 2x^2 - 3x - 5}{h}$$

$$= \lim_{h \to 0} \frac{2[x^2 + 2xh + h^2] + 3x + 3h + 5 - 2x^2 - 3x - 5}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h + \cancel{5} - \cancel{2x^2} - \cancel{3x} - \cancel{5}}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + 3h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h + 3)}{\cancel{h}}$$

$$= 4x + 3 \quad \blacksquare$$

(b) $f(x) = \sqrt{3+2x}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{3+2(x+h)} - \sqrt{3+2x}}{h} \cdot \frac{\sqrt{3+2(x+h)} + \sqrt{3+2x}}{\sqrt{3+2(x+h)} + \sqrt{3+2x}}$$

$$= \lim_{h\to 0} \frac{3+2(x+h)-3-2x}{h\left(\sqrt{3+2(x+h)}+\sqrt{3+2x}\right)}$$

$$= \lim_{h\to 0} \frac{\cancel{3}+\cancel{2x}+2h-\cancel{3}-\cancel{2x}}{h\left(\sqrt{3+2(x+h)}+\sqrt{3+2x}\right)}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{3+2x+2h}+\sqrt{3+2x}\right)}$$

$$= \frac{2}{2\sqrt{3+2x}}$$

$$= \frac{1}{\sqrt{3+2x}} \quad \blacksquare$$

(c) $f(x) = \dfrac{1}{x}$

$$\lim_{h \to 0} \frac{\dfrac{1}{x+h} - \dfrac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{x}{x(x+h)} - \dfrac{x+h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{-h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x^2}$$

26. $h(t) = 58t - 0.83t^2$

The derivative of the position function gives us the velocity function:

$$V(t) = -1.66t + 58$$

(a) $V(1) = -1.66(1) + 58 = 56.34\,m/s$

(b) $V(3) = -1.66(3) + 58 = 53.02\,m/s$

(c) $\text{Average Velocity} = \dfrac{h(3) - h(1)}{2} = \dfrac{166.53 - 57.17}{2}$

$$= \frac{109.36}{2} = 54.68$$

27. (a) $y = x^3 - 2x + 1$, $(-1, 2)$

We first need to find the slope using the derivative:

$$y' = 3x^2 - 2$$

$$y' = 3(-1)^2 - 2 = 3 - 2 = 1$$

Here, $m = 1$.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 1(x + 1)$$

$$y - 2 = x + 1$$

$$y = x + 1 + 2$$

$\therefore$ $y = x + 3$ is the tangent line.

(b) Since the slope of the normal line is the negative reciprical of the tangent line's slope, then we get the following:

$$m = -1$$

$$y - 2 = -1(x+1)$$

$$y - 2 = -x - 1$$

$$y = -x - 1 + 2$$

$\therefore\ y = -x + 1$ is the normal line.

28. $V(t) = 3t^2 - 18t + 15$

(a)

(b) $V(3) = 3(3)^2 - 18(3) + 15$
$= 27 - 54 + 15$
$= -12$ m/s.

(c) When $t = 0$.

(d) $0 = 3(t^2 - 6t + 5)$
$0 = (t-5)(t-1)$
$t = 5 \; \big\} \; t = 1$

+	−	+
	1	5

(e) $a(t) = 6t - 18$

−	+
	3

(f)

+	−	−	+
−	−	+	+
	3	5	

Speeding up: $(1,3) \cup (5,\infty)$.

Slowing down: $(0,1) \cup (3,5)$.

29.

(a) $y = (x^2-4x)(3x^2+5x+2)$

$y' = (x^2-4x)(6x+5) + (3x^2+5x+2)(2x-4)$

$= 6x^3+5x^2-24x^2-20x+6x^3+10x^2+4x-12x^2-20x-8$

$= 12x^3-21x^2-36x-8$ ■

(b) $y = 3\sqrt{x}(2-x^2)$

$y' = 3\sqrt{x}(-2x) + (2-x^2)\cdot 3\cdot \frac{1}{2\sqrt{x}}$

$= \frac{2\sqrt{x}\left[3\sqrt{x}(-2x)\right] + 3(2-x^2)}{2\sqrt{x}}$

$= \frac{-12x^2+6-3x^2}{2\sqrt{x}}$

$= \frac{-15x^2+6}{2\sqrt{x}}$ ■

(c) $y = \sqrt{3x-1}$

$y' = \frac{1}{2}(3x-1)^{-1/2}\cdot 3$

$= \frac{3}{2}\cdot\frac{1}{\sqrt{3x-1}}$

$= \frac{3}{2\sqrt{3x-1}}$ ■

30.

(a) $2x + 3y^2 = 1$

$$2 + 6yy' = 1$$
$$2 - 2 + 6yy' = 1 - 2$$
$$\frac{6yy'}{6y} = \frac{-1}{6y}$$
$$y' = \frac{-1}{6y} \quad \blacksquare$$

(b) $x^3 - 5xy^2 = 2y$

$$3x^2 - [5y^2 + 10xyy'] = 2y'$$
$$3x^2 - 5y^2 = 2y' - 10xyy'$$
$$3x^2 - 5y^2 = y'(2 - 10xy)$$
$$y' = \frac{3x^2 - 5y^2}{2 - 10xy} \quad \blacksquare$$

(c) $y = \dfrac{x^2 + 3}{\sqrt{x}}$

$$y' = \frac{(2x)(\sqrt{x}) - \frac{1}{2\sqrt{x}}(x^2+3)}{(\sqrt{x})^2} = \frac{\frac{4x^2}{2\sqrt{x}} - \frac{(x^2+3)}{2\sqrt{x}}}{x}$$
$$= \frac{\frac{3x^2 - 3}{2\sqrt{x}}}{x} = \frac{3(x^2-1)}{2x\sqrt{x}} \quad \blacksquare$$

31. $f(x) = x^3 + 2x^2 - 4x + 1$

$f'(x) = 3x^2 + 4x - 4$

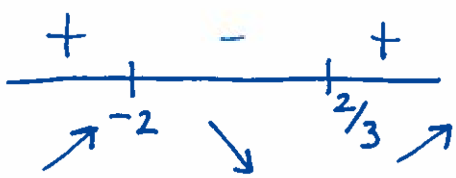
$0 = 3x^2 + 4x - 4$

$0 = (3x-2)(x+2)$

$x = 2/3$ and $x = -2$

$f(-2) = (-2)^3 + 2(-2)^2 - 4(-2) + 1 = 9$

$f(2/3) = (2/3)^3 + 2(2/3)^2 - 4(2/3) + 1 = -13/27$



We have a local minimum at $(2/3, -13/27)$ and a local maximum at $(-2, 9)$.

$f'(-3) = 11 > 0$

$f'(0) = -4 < 0$

$f'(1) = 3 > 0$

The interval of increase is $(-\infty, -2) \cup (2/3, \infty)$.

The interval of decrease is $(-2, 2/3)$.

$f''(x) = 6x + 4$

$0 = 6x + 4$

$\frac{-4}{6} = \frac{6x}{6}$

$x = -2/3$

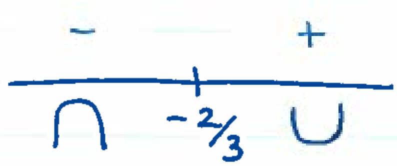
$$f(-2/3) = (-2/3)^3 + 2(-2/3)^2 - 4(-2/3) + 1$$

$$= -\frac{8}{27} + \frac{8}{9} + \frac{8}{3} + 1$$

$$= \frac{-8 + 24 + 72 + 27}{27}$$

$$= \frac{115}{25}$$

$\therefore$ Inflection point at $(-2/3, 115/27)$.



$f''(0) = 4 > 0$, so, concave up $(-2/3, \infty)$

$f''(-1) = -2 < 0$, so, concave down $(-\infty, -2/3)$ ∎