4.5 & 4.6 Solving Radical Equations

When we solve an equation, we "work backwards" to isolate the variable.

Which operation would be "backwards" to finding a square root?

Squaring is the inverse ("backwards") of square rooting!

Squaring will undo square rooting!

Examples: Solving Radicals Involving Square Roots

1. Solve:

a) $\sqrt{3x} = 6$

Notice the variable, *x,* is inside the radical!

There are ***restrictions*** to the values of *x* that can be substituted into the equation.

The value inside the square root must be a ~~positive~~ non-negative number.

Therefore, $3x \geq 0$

which means $x \geq 0$

To solve the radical equation,

we square both sides of the equation and solve for x:

$$\sqrt{3x} = 6$$

Verify:

To solve the radical equation,

we square both sides of the equation and solve for x:

$$\sqrt{3x} = 6$$
$$\left(\sqrt{3x}\right)^2 = (6)^2$$
$$3x = 36$$
$$x = 12$$

Verify:

$$LS\ ?\ RS$$
$$\sqrt{3x} \overset{?}{=} 6$$
$$\sqrt{3 \cdot 12} \overset{?}{=} 6$$
$$\sqrt{36} \overset{?}{=} 6$$
$$6 = 6$$
$$LS = RS$$

Therefore, x = 12 is a solution to the equation.

b. $\sqrt{x+2} = -3$

Restrictions: $x+2 \geq 0$

$x \geq -2$

$\left(\sqrt{x+2}\right)^2 = (-3)^2$ Square both sides of the equation!

$x+2 = 9$

$x = 7$

Need to verify the solution!

LS $\stackrel{?}{=}$ RS

$\sqrt{7+2} \stackrel{?}{=} -3$

$\sqrt{9} \stackrel{?}{=} -3$

$3 \neq -3$

$\therefore$ $x = 7$ is an EXTRANEOUS solution (root)

Thus there is no solution.

b. $\sqrt{x+2} = -3$ Restrictions: $x+2 \geq 0$

$x \geq -2$

$\sqrt{x+2} = -3$

$\left(\sqrt{x+2}\right)^2 = (-3)^2$ Square both sides of the equation!

$x+2 = 9$

$x = 9-2$

$x = 7$

Need to verify the solution!

Verify:

$$LS \; ? \; RS$$
$$\sqrt{x+2} = -3$$
$$\sqrt{7+2} = -3$$
$$\sqrt{9} = -3$$
$$3 \neq -3$$
$$LS \neq RS$$

Since, the LS is NOT equal to the RS, then x = 7 is NOT a solution. In this case, 7 is called an EXTRANEOUS ROOT.

* An Extraneous Root is a solution that does not satisfy the original equation.

Therefore, when solving radical equations, we must verify the solutions by substituting it into the original equation to determine whether it is a solution or whether it is an extraneous root.

c. $\sqrt{x-1}+3=4$ -3 -3 **Restrictions:** $x-1\geq 0$
$x\geq 1$

$\sqrt{x-1}=1$ *Isolate the radical*

$\left(\sqrt{x-1}\right)^2=(1)^2$ *Square both sides of the equation!*

$x-1=1$

$x=2$

Need to verify the solution!

$\sqrt{x-1}+3=4$

$\sqrt{2-1}+3\stackrel{?}{=}4$

$\sqrt{1}+3\stackrel{?}{=}4$

$1+3\stackrel{?}{=}4$

$4=4$ True

$\therefore x=2$ is a good solution

☺

c. $\sqrt{x-1}+3=4$ Restrictions: $x-1\geq 0$

$x\geq 1$

$\sqrt{x-1}+3=4$

$\sqrt{x-1}=4-3$ Isolate the radical

$\sqrt{x-1}=1$

$\left(\sqrt{x-1}\right)^2=(1)^2$ Square both sides of the equation!

$x-1=1$

$x=1+1$

$x=2$

Need to verify the solution!

Verify:

$$LS \, ? \, RS$$
$$\sqrt{x-1}+3=4$$
$$\sqrt{2-1}+3=4$$
$$\sqrt{1}+3=4$$
$$1+3=4$$
$$LS=RS$$

Since, the LS is equal to the RS, then x = 2 is a solution.

Examples: Solving Radicals Involving Cube Roots

If squaring will undo square rooting, what will undo cube rooting?

To solve radicals involving cube roots,

we cube both sides of the equation and solve for x.

1. Solve:

a) $\sqrt[3]{2x-4}=4$

NOTE:
For cube roots, the radicand can be either positive, zero, or negative. Therefore, the equation is defined for all values of x and we state $x\in\Re$.

$\sqrt[3]{2x-4} = 4$ Cube both sides of the equation

$\sqrt[3]{2x-4} = 4$

$\sqrt[3]{2x-4} = 4$

$\left(\sqrt[3]{2x-4}\right)^3 = (4)^3$ Cube both sides of the equation

$2x-4=64$

$2x=64+4$

$2x=68$

$x=34$

Verify:

$$LS\,?\,RS$$
$$\sqrt[3]{2x-4}=4$$
$$\sqrt[3]{2(34)-4}=4$$
$$\sqrt[3]{68-4}=4$$
$$\sqrt[3]{64}=4$$
$$4=4$$
$$LS=RS$$

Since, the LS is equal to the RS, then x = 34 is a solution.

$b)\ \sqrt[3]{x+1} = -3$

$b)\ \sqrt[3]{x+1} = -3$

$$\sqrt[3]{x+1} = -3$$
$$\left(\sqrt[3]{x+1}\right)^3 = (-3)^3$$
$$x+1 = -27$$
$$x = -27-1$$
$$x = -28$$

Verify:

$$LS\ ?\ RS$$
$$\sqrt[3]{x+1} = -3$$
$$\sqrt[3]{-28+1} = -3$$
$$\sqrt[3]{-27} = -3$$
$$-3 = -3$$
$$LS = RS$$

$\therefore x = -28$ is the solution

Your Turn - Don't forget restrictions.

a) $\sqrt{4x} = 8$

$4x \geq 0$

$x \geq 0$

$x = 16$

b) $\sqrt{x+4} = 5$

$x \geq -4$

$x = 21$

c) $7 + \sqrt{2x-3} = 5$

$x \geq \frac{3}{2}$

$x = \frac{7}{2}$ is Extraneous

d) $\sqrt[3]{4x-2} = -1$

$x = \frac{1}{4}$

Your Turn - Don't forget restrictions.

a)
$$\sqrt{4x} = 8 \qquad x \geq 0$$
$$4x = 64$$
$$x = 16$$

b)
$$\sqrt{x+4} = 5 \qquad x \geq -4$$
$$x + 4 = 25$$
$$x = 21$$

$$7 + \sqrt{2x-3} = 5$$
$$\sqrt{2x-3} = -2$$
$$2x - 3 = 4$$
$$2x = 7$$
$$x = \frac{2}{7}$$

c)

$$7+\sqrt{2x-3}=5 \qquad 2x-3\geq 0$$

$$\sqrt{2x-3}=-2 \qquad x\geq \frac{3}{2}$$

$$2x-3=4$$

$$2x=7$$

$$x=\frac{7}{2}$$

$$LS\,?\,RS$$

$$7+\sqrt{2\left(\frac{7}{2}\right)-3}=5$$

$$7+\sqrt{7-3}=5$$

$$7+\sqrt{4}=5$$

$$7+2=5$$

$$9\neq 5$$

$$LS\neq RS$$

Therefore, $x=\frac{7}{2}$ is NOT a solution.

d)

$$\sqrt[3]{4x-2}=-1 \qquad x\in\Re$$

$$\left(\sqrt[3]{4x-2}\right)^3=(-1)^3$$

$$4x-2=-1$$

$$4x=1$$

$$x=\frac{1}{4}$$

$$LS\,?\,RS$$

$$\sqrt[3]{4\left(\frac{1}{4}\right)-2}=-1$$

$$\sqrt[3]{1-2}=-1$$

$$\sqrt[3]{-1}=-1$$

$$-1=-1$$

$$LS=RS$$

Therefore, $x=\frac{1}{4}$ is a solution.

Modelling Real-World Applications

Ex. 1 Collision investigators can approximate the initial velocity, v, in kilometres per hour, of a car based on the length, l, in metres, of the skid mark. The formula $v = 12.6\sqrt{l} + 8, l \geq 0$ models the relationship. What length of skid is expected if a car is travelling 50 km/hr when the brakes are applied? How is knowledge of radical equations used to solve this real world problem?

$V = 12.6\sqrt{l} + 8, \; l \geq 0$

what is l when $v = 50$

$50 = 12.6\sqrt{l} + 8$

$50 - 8 = 12.6\sqrt{l}$

$\frac{42}{12.6} = \frac{12.6\sqrt{l}}{12.6}$

$\sqrt{l} = \frac{42}{12.6}$

$(\sqrt{l})^2 = (3.33)^2$

$l = 11.1\text{ m}$

Modelling Real-World Applications

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$$v = 12.6\sqrt{l} + 8$$
$$50 = 12.6\sqrt{l} + 8$$
$$50 - 8 = 12.6\sqrt{l}$$
$$42 = 12.6\sqrt{l}$$
$$\frac{42}{12.6} = \sqrt{l}$$
$$\left(\frac{42}{12.6}\right)^2 = \left(\sqrt{l}\right)^2$$
$$11.1 = l$$

Ex. 2

The surface area (S) of a sphere with radius r can be found using the equation $S = 4\pi r^2$.

(a) Using the given equation, how could you find the radius of a sphere given its surface area? Write the equation.

(b) The surface area of a ball is 426.2 cm^2. What is its radius?

(a) $\frac{S}{4\pi} = \frac{4\pi r^2}{4\pi} \Rightarrow \sqrt{r^2} = \sqrt{\frac{S}{4\pi}}$

$r = \sqrt{\frac{S}{4\pi}}$

(b) $S = 426.2\text{cm}^2 \Rightarrow r = \sqrt{\frac{426.2}{4\pi}}$

$r = \sqrt{\frac{426.2}{12.566}}$

$r = \sqrt{33.9}$

$r = 5.8$ cm

Ex. 2

The surface area (S) of a sphere with radius r can be found using the equation $S = 4\pi r^2$.

(a) Using the given equation, how could you find the radius of a sphere given its surface area? Write the equation.

(b) The surface area of a ball is 426.2 cm². What is its radius?

a)

$$S = 4\pi r^2$$

$$\frac{S}{4\pi} = r^2$$

$$r = \sqrt{\frac{S}{4\pi}}$$

b)

$$r = \sqrt{\frac{S}{4\pi}}$$

$$r = \sqrt{\frac{426.2}{4\pi}}$$

$$r = 5.8$$

Assign:

p. 222-224, #1ac, 2ac, 4, 6, 8, 11, 12, 15

p. 215 # 1, 2, 3