3.3 Probability using Counting Methods

from worksheet:

A) Yes, all equal

B) No, no positions

C) $_{10}C_3 = \frac{10!}{7!\,3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$

D) $\underbrace{{}_9C_2}_{\text{2 others}} \times \underbrace{1}_{\text{victoria}} = \frac{9!}{7!\,2!} \times 1$

$= \frac{9 \times 8}{2 \times 1} = 36$

E) $P(v) = \frac{36}{120} = \frac{3}{10}$ OR 30%

Ex. 12 students 8 Boys 4 girls

committee of 3

What is probability of having exactly 2 boys 1 girl?

$$P(\text{event}) = \frac{\#\text{ of favorable}}{\text{total}}$$

favorable $\Rightarrow$ ${}_8C_2 \times {}_4C_1 = 28 \times 4 = 112$

total $\Rightarrow$ ${}_{12}C_3 = 220$

$$P(2\text{ boys}, 1\text{ girl}) = \frac{112}{220} = \frac{28}{55}$$

Ex: ① 2 cards from a deck of 52 without replacement.

solution 1:

one king = $\frac{4}{52}$ 2nd King $\frac{3}{51}$

so $P(2K) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$

solution 2: Total # of ways of choosing 2 cards from 52 divided

favorable { by # of ways of choosing 2 Kings from 4 Kings

$$P(2K) = \frac{{}_4C_2}{{}_{52}C_2} = \frac{6}{1326} = \frac{1}{221}$$

Ex ② Group of 10, 4 male, 6 female
Sub-committee of 5 with
EXACTLY 3 female

Solution: Total groups of 5

$$_{10}C_5 = \frac{10!}{(10-5)!\,5!} = \frac{10!}{5!\,5!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 252$$

Favorable: 3 females 2 males

$$_6C_3 \times {}_4C_2 = 120$$

$$P(3f) = \frac{120}{252} = \frac{30}{63} = \frac{10}{21}$$

Ex ③: 5 red, 3 green, 6 blue

3 marbles:

(a) $P(\text{at least one blue}) = \dfrac{\text{favorable } \begin{pmatrix}(3\text{b}),\\(2\text{b, 1other}),\\(1\text{b, 2others})\end{pmatrix}}{\text{total } (3 \text{ from } 14)}$

Total 3 from 14: $_{14}C_3 = 364$

no blue marbles: $_8C_3 = 56$

at least one blue $= 364 - 56 = 308$

$\therefore P(\text{at least one blue}) = \frac{308}{364}$

(B) 1st red, 2nd green, 3rd blue

$P(\text{red}) = \frac{5}{14}$ $P(\text{green}) = \frac{3}{13}$ $P(\text{blue}) = \frac{6}{12}$

$$P(r,g,b) = \frac{5}{14} \times \frac{3}{13} \times \frac{6}{12} = \frac{15}{182} \times \frac{1}{2} = \frac{15}{364}$$

(C) $5 \times 3 \times 6 = 90$ favorable

$_{14}C_3 = 364$

$$P(\text{one r, one g, one b}) = \frac{90}{364} = \frac{45}{182}$$

Practice Questions

7 teachers 3 admin 3 prizes

$$P(T.W.O) = \frac{{}_7P_3}{{}_{10}P_3} = \frac{210}{720} = \frac{21}{72} = \frac{7}{24}$$

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