4.3 Mixed and Entire Radicals

Entire radical is just the radical

Ex: $\sqrt{5}$, $\sqrt{20}$, $\sqrt{48}$, $\sqrt[3]{10}$

Mixed Radical cosists of a number multiplied by a radical

Ex: $2\sqrt{2}$, $2\sqrt{5}$, $3\sqrt{7}$, $\frac{1}{2}\sqrt{10}$, $3\sqrt[4]{25}$

Multiplication Property for Radicals

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

where n is a natural number and a and b are real numbers greater than or = 0

Ex: $\sqrt[3]{9} \cdot \sqrt[3]{3} = \sqrt[3]{9 \cdot 3} = \sqrt[3]{27} = 3$

Ex: $\sqrt{8} \cdot \sqrt{2} = \sqrt{8 \cdot 2} = \sqrt{16} = 4$

rewrite $\sqrt{20}$ as the product of two radicals.

$\sqrt{20} = \sqrt{2} \cdot \sqrt{10}$

$\sqrt{20} = \sqrt{4} \cdot \sqrt{5}$

$= 2\sqrt{5}$

Changing an entire radical to a mixed radical

- Use the multiplication property for radicals to split the radical into the product of two radicals. Make sure one of the factors is a perfect square number, for square roots. A perfect cube for cube roots, etc ...

Ex: write as a mixed radical:

① $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

② $\sqrt{30}$ no perfect square factors, ∴ cannot write as a mixed radical

③ $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

(Always use the biggest perfect "square" factor)

④ $\sqrt[3]{16} = \sqrt[3]{8 \times 2} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}$

⑤ $\sqrt[4]{162} = \sqrt[4]{81 \times 2} = \sqrt[4]{81} \times \sqrt[4]{2} = 3\sqrt[4]{2}$

⑥ $\sqrt{300} = \sqrt{100 \times 3} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$

⑦ Simplify $2\sqrt{50} = 2\sqrt{25 \times 2}$

$= 2 \times \sqrt{25} \times \sqrt{2}$

$= 2 \times 5 \times \sqrt{2}$

$= 10\sqrt{2}$

⑧ $\frac{1}{2}\sqrt{40} = \frac{1}{2}\sqrt{4 \times 10} = \frac{1}{2}\sqrt{4} \times \sqrt{10}$

$= \frac{1}{2}(2)\sqrt{10}$

$= 1\sqrt{10}$

$= \sqrt{10}$

Changing to a mixed radical using prime factorization

Ex: $\sqrt{28}$

$= \sqrt{2 \times 2 \times 7}$

$= \sqrt{2^2 \cdot 7}$

$= 2\sqrt{7}$

28
2 × 14
2 × 2 × 7

Ex② $\sqrt[3]{40}$

$= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5}$

$= \sqrt[3]{2^3 \cdot 5}$

$2\sqrt[3]{5}$

40
2 × 20
2 × 10
2 × 5

Ex③ $\sqrt[5]{486}$

$\sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 2}$

$\sqrt[5]{3^5 \cdot 2}$

$3\sqrt[5]{2}$

486
2 × 243
3 × 81
3 × 27
3 × 9
3 × 3

Switch from a mixed radical to an entire radical.

Ex① $5\sqrt{3}$

$= \sqrt{3 \cdot 5^2}$

$= \sqrt{3 \cdot 25}$

$= \sqrt{75}$

Multiply the radicand by the rational "coefficient" raised to the power of the index.

Ex② $2\sqrt[3]{5} = \sqrt[3]{5 \cdot 2^3} = \sqrt[3]{5 \cdot 8} = \sqrt[3]{40}$

Ex③ $10\sqrt{7} = \sqrt{7 \cdot 10^2} = \sqrt{700}$

Ex④ $5\sqrt[4]{3} = \sqrt[4]{3 \cdot 5^4} = \sqrt[4]{3 \cdot 625} = \sqrt[4]{1875}$

Practice: page 218 # 3-6, 9-13, 17, 18
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Assess your understanding page 221 # 1-11