2.2 Permutations and factorial Notation

Factorial symbol is !

usually written as n! on a calculator

This means to multiply all of the digits from the number given, n, down to 1 (NOTE: only works for non-negatives)

Ex: $3! = 3 \times 2 \times 1 = 6$

Ex: $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Ex: $\frac{23!}{20!} = \frac{23 \times 22 \times 21 \times \cancel{20!}}{\cancel{20!}} = 23 \times 22 \times 21 = 10626$

Permutations: An arrangement of distinguishable objects in a definite order.

Ex: The letters A and B have two permutations AB and BA

Ex: How many ways can you arrange 6 students in a row of 6 seats?

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Ex: There are 4 boys and 2 girls to be seated in a row of 6 seats. How many ways can these students be seated if a girl must sit at each end of the row?

$2 \times 4 \times 3 \times 2 \times 1 \times 1 = 48$ ways

(ii) if a boy must sit at each end?

$4 \times 4 \times 3 \times 2 \times 1 \times 3 = 288$ ways

(iii) if the two girls must sit together?

$G_1\ G_2\ 4\ 3\ 2\ 1 = 5!$

$\_\ G_1\ G_2\ \_\ \_\ \_$

$\_\ \_\ G_1\ G_2\ \_\ \_$

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$\_\ \_\ \_\ \_\ G_1\ G_2$

There are 5 ways the girls can move across the row, there are 4! ways of arranging the boys and there are 2! ways of arranging the girls seated together

So, in total there are $5 \times 4! \times 2!$

$= 5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1$

$= 240$

(*)

Ex: How many ways can you arrange 3 boys and 5 girls if the ~~boys~~ must sit together, in 8 seats?

Solution: If we think of the 3 boys as a single seat then there 6! ways of arranging them with the 5 girls. We must also multiply by the 3! arrangements of the ~~boys~~ within their group.

So, in total there are $6! \times 3!$

$= 720 \times 6 = 4320$

Ex: How many permutations are there using the letter of the word MATH?

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$

Ex: write $5 \times 4 \times 3$ using factorial notation.

$$\frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}}$$

Ex: Solve: $\frac{(n+1)!}{n!} = 6$

$$\frac{(n+1)!}{n!} = \frac{(n+1)(n)(n-1)(n-2)\cdots 3\times 2\times 1}{n(n-1)(n-2)\cdots 3\times 2\times 1}$$

$$\frac{(n+1)!}{n!} = \frac{(n+1)n!}{n!} = n+1$$

So $n+1 = 6$

$n = 5$

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