UNIT 2: (Ch 1) Transformations on functions

1.1 Horizontal and Vertical Translations

Investigate how $y - k = f(x)$ changes $y = f(x)$

Ex: compare $y - 3 = x^2$ to $y = x^2$

to graph $y - 3 = x^2$ rewrite as $y = x^2 + 3$

Vertical Translation of 3 units up (VT)

Ex: compare $y + 2 = x^2$ to $y = x^2$

$y - (-2) = x^2$

graph $y = x^2 - 2$

V.T. of 2 units down

Ex: compare $y - 2 = \sin x$ to $y = \sin x$

V.T. of 2 units up

In general $y - k = f(x)$ will change $y = f(x)$ by a V.T. of K units

In a mapping rule we say

$(x, y) \rightarrow (x, y + k)$

all points (x, y) are mapped (changed) to new points $(x, y + k)$

Investigate how $y = f(x-h)$ changes $y = f(x)$

Ex: compare $y = (x-3)^2$ to $y = x^2$

Horizontal Translation of 3 right

Compare $y = (x+5)^2$ to $y = x^2$

$y = (x-(-5))^2$

H.T. of 5 left

So, $y = f(x-h)$ will change $y = f(x)$ by a H.T. of h units

Mapping Rule: $(x, y) \rightarrow (x+h, y)$

What about $y-k=f(x-h)$?

mapping: $(x,y) \rightarrow (x+h, y+k)$

Ex: Given $y=x^2$, how does

$(x,y) \rightarrow (x-2, y+3)$ change

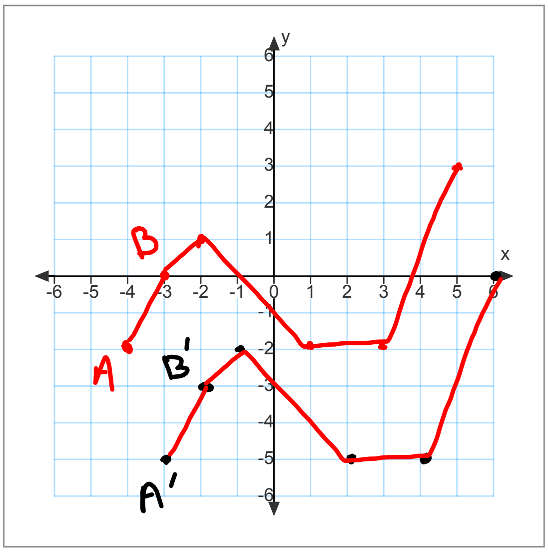
the graph and equation?

Solution: Graph has moved 2 units left

and 3 units up

Equation is: $y-3=(x+2)^2$

Ex: Sketch the new graph according to the transformations



$(x,y) \rightarrow (x+1, y-3)$

$(-4,-2) \rightarrow (-4+1, -2-3)$

$(-3,-5)$

$(-3,0) \rightarrow (-2,-3)$

$(-2,1) \rightarrow (-1,-2)$

$(1,-2) \rightarrow (2,-5)$

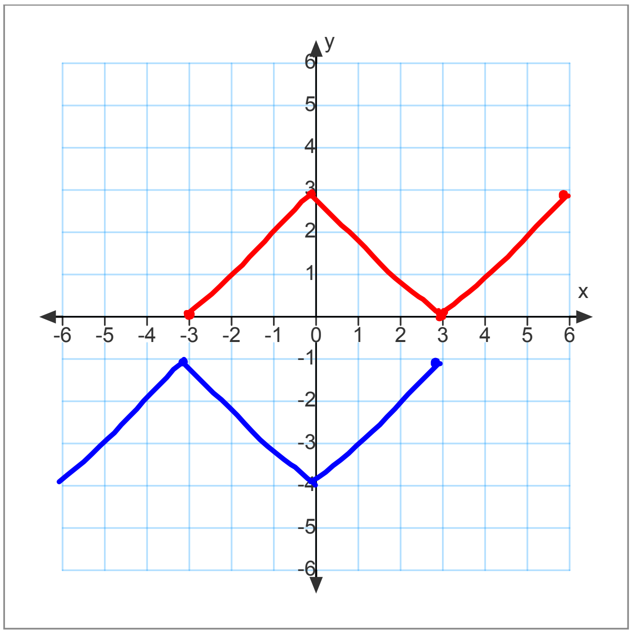
$(3,-2) \rightarrow (4,-5)$

$(5,3) \rightarrow (6,0)$

$y+3=f(x-1)$

$(x, y) \rightarrow (x-3, y-4)$

$y+4 = f(x+3)$



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#'s 1-5, 8, 9, 10, 11,

1.2 Reflections and stretches

Reflection in x-axis:

change from $y = f(x)$ to $-y = f(x)$

in mapping notation: $(x, y) \rightarrow (x, -y)$

Reflection in y-axis:

Change from $y = f(x)$ to $y = f(-x)$

in mapping notation $(x, y) \rightarrow (-x, y)$

$y = a\,f(x)$

Vertical Stretch:

$y = f(x)$ changed to $\frac{1}{a}y = f(x)$

is a vertical stretch by a factor of "a"

mapping notation: $(x, y) \rightarrow (x, ay)$

Horizontal Stretch:

$y = f(x)$ changed to $y = f(bx)$ is a horizontal stretch by a factor of $\frac{1}{b}$

mapping notation: $(x, y) \rightarrow (\frac{1}{b}x, y)$

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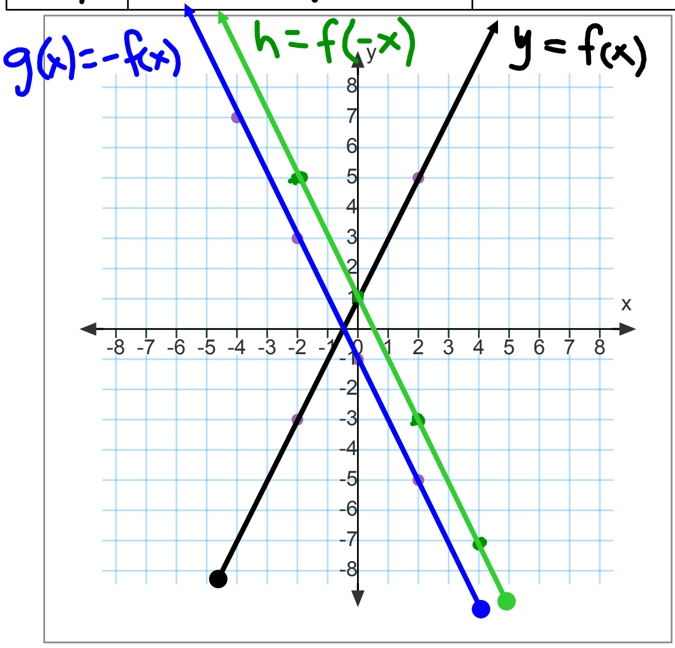
(a)

x	$f(x) = 2x+1$	$g(x) = -f(x)$ reflection in x-axis	$h(x) = f(-x)$ Reflection in y-axis
-4	$2(-4)+1 = -7$	7	$2(4)+1 = 9$
-2	$2(-2)+1 = -3$	3	$2(2)+1 = 5$
0	1	-1	1
2	5	-5	$2(-2)+1 = -3$
4	9	-9	-7



$h(x) = 2(-x)+1$

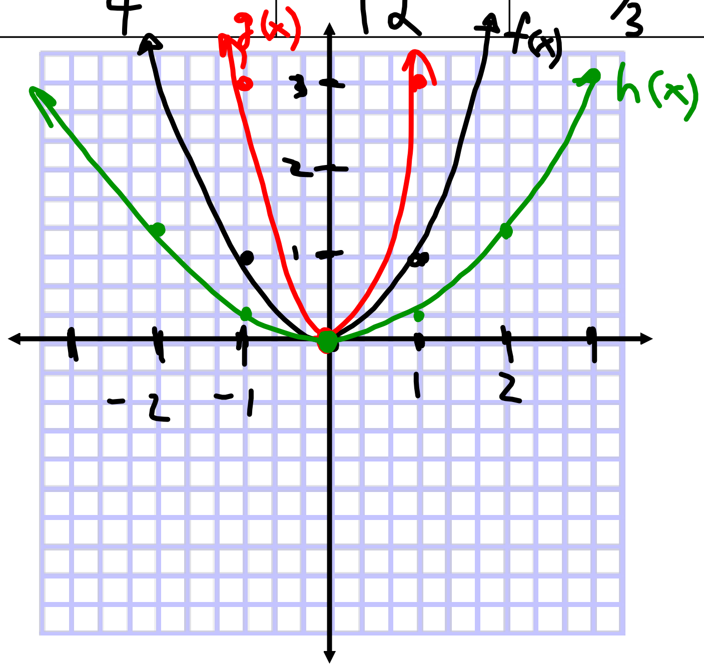
$h(x) = -2x+1$

$g(x) = -(2x+1)$

$g(x) = -2x-1$

#2

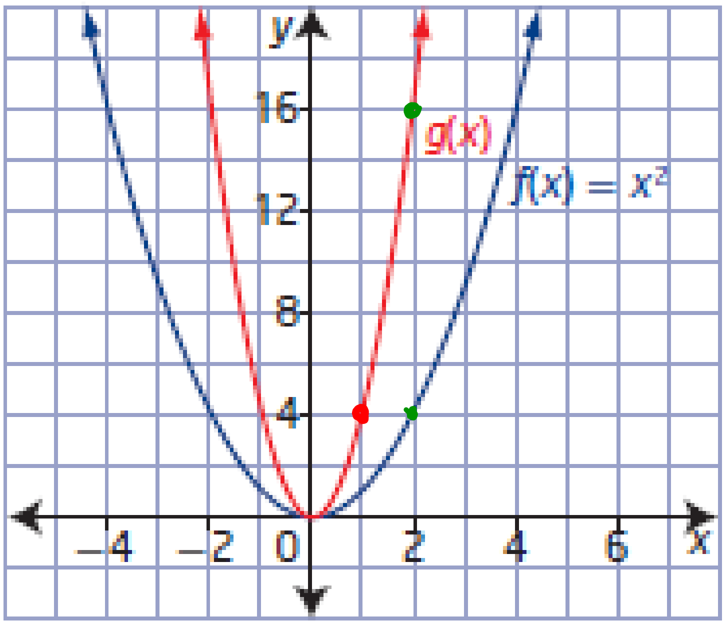
x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
-2	4	12	$4/3$
-1	1	3	$1/3$
0	0	0	0
1	1	3	$1/3$
2	4	12	$4/3$



$y = x^2$

x	y
2	4

x	y
1	4



x	y
2	16

$(x,y) \to (\frac{1}{2}x, y)$

$y = (2x)^2$

$(x,y) \to (x, 4y)$

$y = 4x^2$

$\frac{1}{4}y = x^2$

1.3 Combining Transformations

when performing multiple transformations, the reflections and stretches should be performed before any translations. (RST)

$y = f(x)$ becomes $y - k = af\left(\frac{1}{b}(x-h)\right)$

OR $y = af\left(\frac{1}{b}(x-h)\right) + k$

$(x, y) \rightarrow (bx + h, ay + k)$

Ex: Describe the transformations

① $y = -3f(2(x+1))$

- Reflection in x-axis
- V.S. by a factor of 3
- H.S. by a factor of $\frac{1}{2}$
- H.T. by 1 unit left

$(x, y) \rightarrow (\frac{1}{2}x - 1, -3y)$

(x,y) ⇒ (-3x, 1/2y + 5)

② $y = \frac{1}{2}f(-\frac{1}{3}x) + 5$

- Reflection in y-axis
- V.S. by a factor of $\frac{1}{2}$
- H.S. by a factor of 3
- V.T. of 5 units up

$(x, y) \rightarrow (-3x, \frac{1}{2}y + 5)$

Ex: compare $y = f(x)$ to $y = f(2(x-1))$

if $f(x) = x^2$

$$y = f(2(x-1)) = (2(x-1))^2$$

$(x, y) \rightarrow (\frac{1}{2}x + 1, y)$ HS by $\frac{1}{2}$

HT of 1 right

Ex: Given that $y = f(x)$ has been transformed by a V.S. by 2, a H.S. by $\frac{1}{4}$, a reflection in both the x-axis and y-axis, a VT of 5 down and a HT of 7 left, what is the equation?

Solution: $(x, y) \rightarrow (-\frac{1}{4}x - 7, -2y - 5)$

$$y = -2 \cdot f(-4(x+7)) - 5$$

1.3 Practice Questions

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\# 1-4, 6, 7, 8, 9, 10

Invariant Points :

Points which remain same after Transformation.

Ex: If a function $f(x)$ has domain: $\{x \mid -4 \le x \le 4, x \in \mathbb{R}\}$ and Range : $\{y \mid 0 \le y \le 4, y \in \mathbb{R}\}$, what are the domain and range of $g(x)$ if $g(x) = f(2x)$?

Solution: write the mapping rule and apply it to the values in domain and range.

$$(x, y) \longrightarrow \left(\tfrac{1}{2}x, y\right)$$

Domain of $g(x) = \{x \mid \frac{1}{2}(-4) \le x \le \frac{1}{2}(4), x \in \mathbb{R}\}$

$= \{x \mid -2 \le x \le 2, x \in \mathbb{R}\}$

Range of $g(x) = \{y \mid 0 \le y \le 4, y \in \mathbb{R}\}$

Ex: what about $h(x)$, if $h(x) = -\frac{1}{2}f(x)$

$$(x, y) \to \left(x, -\tfrac{1}{2}y\right)$$

Domain = Same as $f(x)$

Range: $\{y \mid -\frac{1}{2}(0) \ge y \ge -\frac{1}{2}(4), y \in \mathbb{R}\}$

$\{y \mid -2 \le y \le 0, y \in \mathbb{R}\}$

The graph of $y = f(x)$ with points $A(5, 3)$, $B(3, 6)$, $C(-1, -3)$ is transformed so that $A'(-9, -1)$, $B'(-5, 0)$, $C'(3, -3)$. Plot the points and determine the equation of the image function in the form $y = af(b(x-h))+k$.

x_2 y_2

$(5, 3) \rightarrow (-9, -1)$

x_1 y_1

$(3, 6) \rightarrow (-5, 0)$

$(-1, -3) \rightarrow (3, -3)$