3.2 Perfect squares, Perfect cubes and their Roots

perfect square: a number which can be written as another number squared. (the product of a number times itself)

Ex:

$1 = 1^2$ $\quad 25 = 5^2$

$4 = 2^2$ $\quad 36 = 6^2$

$9 = 3^2$ $\quad 49 = 7^2$

$16 = 4^2$ $\quad 64 = 8^2$

Determining Square roots using prime factorization.

what is the prime factorization of:

36

2 × 18

2 × 2 × 9

2 × 2 × 3 × 3

$\boxed{2\times3}\times\boxed{2\times3}$

100

2 × 50

2 × 2 × 25

2 × 2 × 5 × 5

$\boxed{2\times5}\times\boxed{2\times5}$

Ex:

324

2 × 162

2 × 2 × 81

2 × 2 × 9 × 9

2 × 2 × 3 × 3 × 3 × 3

$\boxed{2\times3\times3}\times\boxed{2\times3\times3}$

$18 \times 18 = 18^2$

so $18^2 = 324$

and $\sqrt{324} = 18$

Is 576 a perfect square?

576

2×288

$2 \times 2 \times 144$

$2 \times 2 \times 12 \times 12$

$2 \times 2 \times 2 \times 6 \times 2 \times 6$

$2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 3$

$\boxed{2 \times 2 \times 2 \times 3} \times \boxed{2 \times 2 \times 2 \times 3}$

24×24

so $24^2 = 576$ yes 576 is a perfect square!

Is 1225 a perfect square?

1225

5×245

$5 \times 5 \times 49$

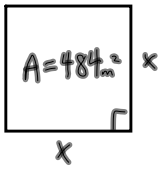
$5 \times 5 \times 7 \times 7$

$= \boxed{5 \times 7} \times \boxed{5 \times 7} = 35 \times 35$.

so $1225 = 35^2$

yes 1225 is a perfect square!

Ex:



Area $= x^2$

$484 = x^2$

484

2×242

$2 \times 2 \times 121$

$2 \times 2 \times 11 \times 11$

$2 \times 11 \times 2 \times 11$

$22^2 = 484$

so $\boxed{x = 22}$

Perfect Cube :

A number which can be written as the product of another number times itself times itself. Another number cubed.

Ex:

$2 \times 2 \times 2 = 8$

$3 \times 3 \times 3 = 27 = 3^3$

$4 \times 4 \times 4 = 64 = 4^3$

So, 8, 27 and 64 are examples of perfect cubes.

Checking for perfect cubes and cube roots using prime factorization.

Ex: write 1000 using prime factorization

1000

2 × 500

2 × 2 × 250

2 × 2 × 2 × 125

2 × 2 × 2 × 5 × 25

2 × 2 × 2 × 5 × 5 × 5

$2 \times 5 \times 2 \times 5 \times 2 \times 5$

$10 \times 10 \times 10$

10^3

since $1000 = 10^3$

1000 is a perfect cube!

Ex: Is 1728 a perfect cube?

1728

2 × 864

2 × 2 × 432

2 × 2 × 2 × 216

2 × 2 × 2 × 2 × 108

2 × 2 × 2 × 2 × 2 × 54

2 × 2 × 2 × 2 × 2 × 2 × 27

2 × 2 × 2 × 2 × 2 × 2 × 3 × 9

3 × 3 × 3

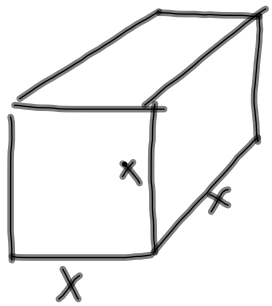
$\boxed{2 \times 2 \times 3}$ $\boxed{2 \times 2 \times 3}$ $\boxed{2 \times 2 \times 3}$

$12 \times 12 \times 12$

OR

12^3

Ex: The volume of a cube is $2744\ cm^3$ what is the length of one of its sides?



$V = l \times w \times h$

$V = x \cdot x \cdot x$

$V = x^3$

$2744 = x^3$

cube root

$\sqrt[3]{}$

2744

2 × 1372

2 × 2 × 686

2 × 2 × 2 × 343

2 × 2 × 2 × 7 × 49

2 × 2 × 2 × 7 × 7 × 7

(2×7) × (2×7) × (2×7)

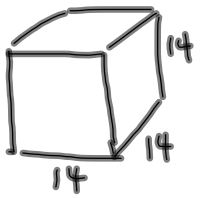
$= 14 \times 14 \times 14$

OR

$14^3 = 2744$

So the length of each side is 14 cm

What is the area of one side?



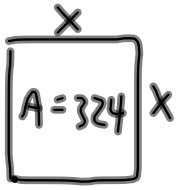
$A = 14 \times 14 = 196\ cm^2$

What is the total surface area of the cube?

one side is $196\ cm^2$ and a cube has 6 congruent sides, therefore the total surface area is $6 \times 196 = 1176\ cm^2$

Ex: The total surface area of a cube is 1944 cm^2, what is the volume?

Solution: one side $= 1944 \div 6 = 324$

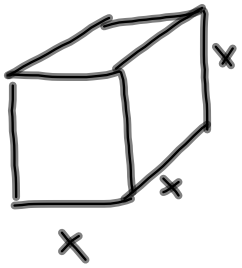


$x^2 = 324$

$x = \sqrt{324}$

$x = 18$

324
2×162
$2 \times 2 \times 81$
$2 \times 2 \times 3 \times 3 \times 3 \times 3$
$2 \times 3 \times 3 \times 2 \times 3 \times 3$



$V = 18 \times 18 \times 18$

$V = 18^3$

$V = 5832$

p. 146 -147 # 4-8, 10, 11, 13.

p. 149 # 1, 2, 3, 6, 7, 8, 10

Square Root and cube root notation

Square root of 25 $\Rightarrow \sqrt{25}$

"what number squared = 25"

$\sqrt{25} = 5$ because $5^2 = 25$

Cube root of 27 $\Rightarrow \sqrt[3]{27}$

"what number cubed = 27"

$\sqrt[3]{27} = 3$ because $3^3 = 27$

Ex: $\sqrt[3]{125} = 5$