2.4. Permutations when Objects are Identical

Ex: How many unique arrangments can be made with the letters GLEE ? (12) $\frac{4!}{2!} = \frac{24}{2}$

G L E E	L E E G
L G E E	E E G L
G E L E	E E L G
G E E L	E L E G
L E G E	E G E L
E L G E	E G L E

If there are n objects where there are "a" repeats of one object and "b" repeats of a second object and "c" repeats of a third object, etc... then there are

$\frac{n!}{a!b!c!}$ distinct arrangements.

Ex: How many different ways can the letters of CANADA be arranged?

Solution: $\frac{6!}{3!} = 6\times5\times4 = 120$

Ex: How many different arrangements can be made with the letters MISSISSIPPI ?

Solution: $\frac{11!}{4!4!2!} = \frac{11\times10\times9\times8\times7\times6\times5\times4!}{4!\,4!\,2!}$

$= \frac{11\times10\times9\times8\times7\times6\times5}{4\times3\times2\times1\times2\times1}$

$= 34650$

Ex: How many ways can you arrange the letters of CANADA if the 1st letter is a C and the last letter is N?

Solution: C _ _ _ _ N

$\frac{4!}{3!} = 4$

$= 1 \times 4 \times 1$

$= 4$

$$\frac{n!}{a!\,b!\,c!}$$

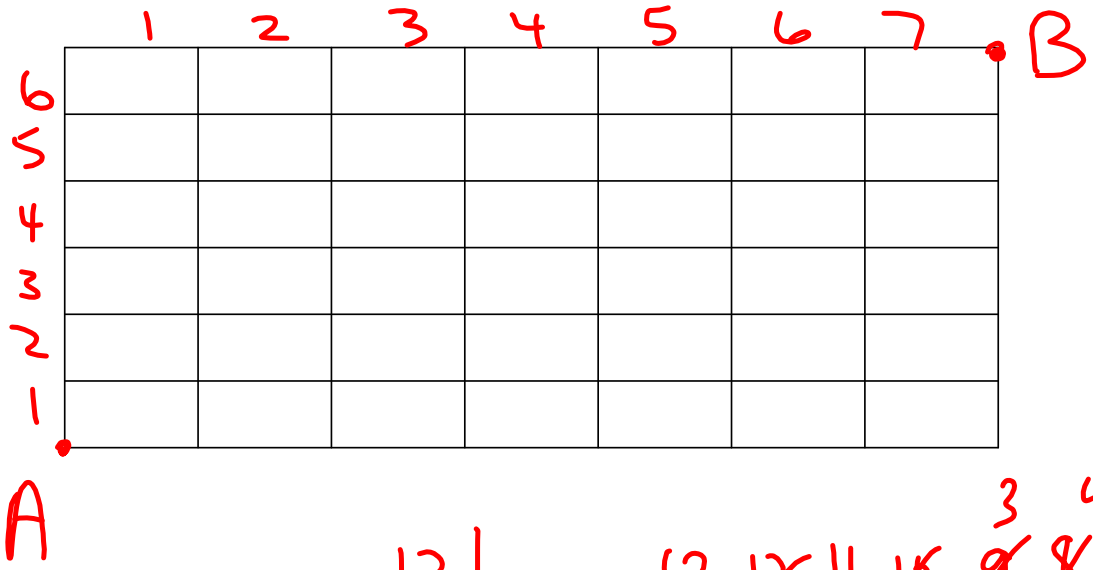
Practice: P. 104 - 107
#'s 1-6, 12, 15, 16, 17

A E E E E E

S S S S B

$$\frac{9!}{4!\,5!} = \frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1} = 126$$

3



$$\frac{13!}{6!\,7!} = \frac{13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$$

$$= 1716$$