2.5 Combinations

Permutation : order matters
AB and BA are two permutations

Combination : order is not important
AB and BA is only one combination

Ex: Think about the letters A, B, C

Permutations of 2 letters : 6 permutations

AB AC BA BC CA CB

Combinations of 2 letters : 3 combinations

AB AC BC

2.6 Working with Combinations

Paige Heather Justin Andrew
Courtney Morgan Tom

Pick a team of 3

T C H
C T H
T H C
C H T
H T C
H C T

Same combination

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

$${}_7C_3 = \frac{7!}{(7-3)!\,3!}$$

Ex: Choose 6 numbers at random from 1 to 49

$(49!) \div (43! \times 6!)$

$$ {}_{49}C_6 = \frac{49!}{(49-6)!\,6!} $$

$43!$

$$ = \frac{49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43 \cdots}{(43 \cdot 42 \cdots)(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} $$

$$ = \frac{49 \cdot 48 \cdot 47 \cdot 46 \cdot \overset{3}{45} \cdot 44}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} $$

$$ = 49 \cdot 47 \cdot 46 \cdot 3 \cdot 44 $$

$$ = 13\,983\,816 $$

$$ {}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n!}{r!\,(n-r)!} $$

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2. From a group of five students, three students need to be chosen for a car-wash committee.

a) How many committees are possible? ${}_5C_3 = 10$

b) How many committees are possible, if only two students are needed on the committee? ${}_5C_2 = 10$

c) Compare your answers for parts a) and b). What do you notice? Explain why this occurred.

(a) $${}_5C_3 = \frac{5!}{(5-3)!\,3!} = \frac{5!}{2!\,3!} = \frac{5\cdot 4\cdot \cancel{3!}}{2!\,\cancel{3!}} = \frac{20}{2} = 10$$

(b) $${}_5C_2 = \frac{5!}{(5-2)!\,2!} = \frac{5!}{3!\,2!} = 10$$

3. How many ways can 6 people be selected from a group of 12 to form a dodge-ball team?

ANS: ${}_{12}C_6 = 924$ $\quad \frac{12!}{(12-6)!\,6!}$

18. There are 7 boys and 13 girls in the school art club. A group of 5 is needed to set up an art exhibit. How many different groups of 5 students with at least 2 boys are there to choose from?

a) Solve the problem using direct reasoning.

b) Solve the problem using indirect reasoning.

c) Which approach do you prefer? Explain why.

boys

At least 2 boys ⇒ 2 boys 3 girls = $_7C_2 \times {}_{13}C_3 = 6006$

3 boys 2 girls = $_7C_3 \times {}_{13}C_2 = 2730$

4 boys 1 girl = $_7C_4 \times {}_{13}C_1 = 455$

5 boys 0 girls = $_7C_5 \times {}_{13}C_0 = 21$

IN TOTAL = 9212

Less than 2 boys ⇒ 1 boy 4 girls = $_7C_1 \times {}_{13}C_4 = 5005$

0 boys 5 girls = $_7C_0 \times {}_{13}C_5 = 1287$

6292

$_{20}C_5 = 15504$

$15504 - 6292 = 9212$

$_6C_2$

$$\binom{6}{2} = \binom{6}{4}$$ ✓

$$\frac{6!}{(6-2)!\,2!} \qquad \frac{6!}{(6-4)!\,4!}$$

$$\frac{6!}{4!\,2!} \qquad \frac{6!}{2!\,4!}$$

13) (a) (i) $_nC_r = \frac{5!}{3!(5-3)!}$

$n = 5$

$r = 3$

8.) 15 players choose 9 if Connie is the pitcher.

(a) Permutation b/c positions imply order matters.

(b) $_{14}P_8 = \frac{14!}{(14-8)!} = \frac{14!}{6!} = 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

$=$

$${}_8C_3 = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1}$$

$$\binom{8}{3} = 56$$

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#4. four card hands with one card from each suit

S C D H

$13 \times 13 \times 13 \times 13 = 28561$

#5 200 tickets top 5 cash prizes

(a) if tickets are not replaced

200 199 198 197 196 = 304 278 004 800

(b) if tickets are replaced

200 200 200 200 200

= 320 000 000 000

12 3 numbers from 0 to 99

All $100 \times 100 \times 100 = 1\,000\,000$

ODD $50 \times 50 \times 50 = 125\,000$

Extra practice:

① From a standard deck of 52 cards how many 5 card hands have:

(a) at least one red card

R B B B B $\quad {}_{26}C_1 \times {}_{26}C_4$ 388700

R R B B B $\quad {}_{26}C_2 \times {}_{26}C_3$ 845000

R R R B B $\quad {}_{26}C_3 \times {}_{26}C_2$ 845000

R R R R B $\quad {}_{26}C_4 \times {}_{26}C_1$ 388700

R R R R R $\quad {}_{26}C_5$ 65780

$${}_{52}C_5 - {}_{26}C_5$$

$$\frac{52!}{(52-5)!\,5!} - \frac{26!}{(26-5)!\,5!}$$

$$\frac{52!}{47!\,5!} - \frac{26!}{21!\,5!}$$

$$2\,598\,960 - 65\,780$$

$$2\,533\,180$$

(b) at least one face card (J, Q, or K)

F	N	N	N	N	$_{12}C_1 \times {}_{40}C_4$
F	F	N	N	N	$_{12}C_2 \times {}_{40}C_3$
F	F	F	N	N	$_{12}C_3 \times {}_{40}C_2$
F	F	F	F	N	$_{12}C_4 \times {}_{40}C_1$
F	F	F	F	F	$_{12}C_5$

OR $_{52}C_5 - {}_{40}C_5$