4.4 Simplifying Algebraic Expressions Involving Radicals.

Ex: $\sqrt{x}$, $\sqrt{x^2}$, $\sqrt[3]{x^4}$, $\sqrt{x^3y^4}$

Question: Is $\sqrt{x}$ defined for all values of x?

Answer: No $\sqrt{x}$ is only defined for $x \geq 0$

This is called the Restrictions on the variable x.

Restrictions: tell us what values we can replace the variable with so that we get a real answer.

Ex: what are the restrictions on x for $\sqrt{x^2}$? "Everything works"

$x \in R$

Note: $\sqrt{x^2} = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

OPPOSITE

Note: $|x|$ is called the absolute value of x and is defined as the distance (always positive) from x to zero on a number line.

Ex: $|3| = 3$, $|5| = 5$

$|-2| = 2$ $|-1| = 1$

We can say that $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

Recall that $\sqrt{x^2} = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

$\therefore$ we can say that $\boxed{\sqrt{x^2} = |x|}$

We can use this to help simplify radical expressions involving powers of x.

Ex: State the restrictions on x, and simplify completely:

① $\sqrt{x^3}$, $x \geq 0$

$\sqrt{x^2 \cdot x}$

$|x|\sqrt{x}$

$x\sqrt{x}$

Since $x \geq 0$ $|x| = x$

$\sqrt{27}$

$\sqrt{3^2 \cdot 3}$

$3\sqrt{3}$

$\sqrt{x^2} = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

② $\sqrt{8x^5}$, $x \geq 0$

$\sqrt{4 \cdot 2x^2 \cdot x^2 \cdot x}$

$2x \cdot x\sqrt{2x}$

$2x^2\sqrt{2x}$

③ $\sqrt{x+2}$

Restrictions:

Radicand ≥ 0

$x + 2 \geq 0$

$x + \cancel{2 - 2} \geq 0 - 2$

$x \geq -2$

$\sqrt{x+2}$

No factors,

$\therefore$ completely simplified

④ $\sqrt{x^2+4}$

Restrictions: $x^2 + 4 \geq 0$

$x^2 + \cancel{4 - 4} \geq 0 - 4$

$x^2 \geq -4$

true for all $x \in \mathbb{R}$

$\therefore$ $\sqrt{x^2+4}$ works for $x \in \mathbb{R}$

$\sqrt{x^2+4}$

No factors

$\therefore$ completely Simplified

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#1, 2, 3, 11

Adding/subtracting Radicals with algebraic expressions

Just like earlier, we must have like radicals, same index and same radicand.

In addition, we must have like algebraic expressions "in front" of the radical. Just like when adding/subtracting algebraic expressions alone.

Like Terms	not like terms
x^2 and $3x^2$	x^2 and $3x^4$
$\sqrt{2x}$ and $5\sqrt{2x}$	$\sqrt{2x}$ and $5\sqrt{2}$
$-3x\sqrt{2}$ and $4x\sqrt{2}$	$-3x\sqrt{2}$ and $4x^2\sqrt{2}$
$3x\sqrt{2}$ and $2x\sqrt{2}$	$\sqrt{2x}$ and $\sqrt{3x}$
	$3x\sqrt{2}$ and $2\sqrt{2}$
	$3x\sqrt{2}$ and $3\sqrt{2}$

Ex: Simplify

① $3x\sqrt{2} + 5x\sqrt{2} = 8x\sqrt{2}$

$\sqrt{x^n}$

if n is even $x \in \mathbb{R}$

if n is odd $x \geq 0$

② $5x\sqrt{3} - \sqrt{12x^3}$

Note: restrictions $x \geq 0$

$5x\sqrt{3} - \sqrt{4 \cdot 3 \cdot x^2 \cdot x}$

$5x\sqrt{3} - 2x\sqrt{3x}$

not like terms!
we are finished!

③ $2\sqrt{18x^2} + 5\sqrt{32x^2}$

Note: restrictions $x \in \mathbb{R}$

$2\sqrt{9 \cdot 2x^2} + 5\sqrt{16 \cdot 2x^2}$

$2 \cdot 3|x|\sqrt{2} + 5 \cdot 4|x|\sqrt{2}$

$6|x|\sqrt{2} + 20|x|\sqrt{2}$

$= 26|x|\sqrt{2}$

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1, 2, 3, 5, 6a, 11
4, 6(b, c, d), 8, 9
10, 12, 15

Multiplying Algebraic Expressions with radicals

Ex: $\left(5\sqrt{6x^2}\right)\left(-2x\sqrt{2x}\right)$ is true when $x \geq 0$, $x \in \mathbb{R}$

$$(5)(-2x)\sqrt{(6x^2)(2x)}$$

$$-10x\sqrt{12x^3}$$

$$-10x\sqrt{4 \cdot 3 x^2 \cdot x}$$

$$-10 \cdot 2x \cdot x\sqrt{3x}$$

$$-20x^2\sqrt{3x}$$

Ex ② $\left(\sqrt{80x^3}\right)\left(\sqrt{20x^2}\right)$ Restrictions $x \geq 0$, $x \in \mathbb{R}$

$$\left(\sqrt{16 \cdot 5x^2 x}\right)\left(\sqrt{4 \cdot 5x^2}\right)$$

$$\left(4x\sqrt{5x}\right)\left(2x\sqrt{5}\right)$$

$$(4x)(2x)\sqrt{(5x)(5)}$$

$$8x^2\sqrt{25x}$$

$$8 \cdot 5x^2\sqrt{x}$$

$$40x^2\sqrt{x}$$

Ex ③ $(2\sqrt{x}+3)(5-3\sqrt{x}) \quad x \geq 0, x \in \mathbb{R}$

$2\sqrt{x}(5-3\sqrt{x})+3(5-3\sqrt{x})$

$10\sqrt{x}-6\sqrt{x^2}+15-9\sqrt{x}$

$10\sqrt{x}-9\sqrt{x}-6\sqrt{x^2}+15$

$\sqrt{x}-6x+15$

Ex ④ $5\sqrt{42x}(4+4\sqrt{7x}) \quad x \geq 0, x \in \mathbb{R}$

$5\cdot4\sqrt{42x}+5\cdot4\sqrt{42\cdot7x^2}$

$20\sqrt{42x}+20\sqrt{6\cdot7\cdot7x^2}$

$20\sqrt{42x}+20\cdot7x\sqrt{6}$

$20\sqrt{42x}+140x\sqrt{6}$

Dividing Algebraic expressions with radicals

Ex: $\dfrac{15\sqrt{x^3}}{3\sqrt{x^2}}$ $x > 0,\ x \in R$

$$\boxed{\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}}$$

$5\sqrt{\dfrac{x^3}{x^2}}$

$5\sqrt{x}$

Ex② $\dfrac{\sqrt{5x^4} + 3\sqrt{2x}}{\sqrt{3x^3}}$ $x > 0,\ x \in R$

$\sqrt{x^4}$

$\sqrt{x^2 \cdot x^2}$

$x \cdot x$

$\dfrac{\left(x^2\sqrt{5} + 3\sqrt{2x}\right)}{\left(x\sqrt{3x}\right)} \dfrac{\left(\sqrt{3x}\right)}{\left(\sqrt{3x}\right)}$ Rationalize the denominator

$\dfrac{x^2\sqrt{15x} + 3\sqrt{6x^2}}{x\sqrt{9x^2}}$

$\dfrac{x^2\sqrt{15x} + 3x\sqrt{6}}{x(3x)}$

$\dfrac{x^{\cancel{2}}\sqrt{15x} + 3\cancel{x}\sqrt{6}}{3x^{\cancel{2}}}$

Reduce each by a factor of x

$$\boxed{\frac{x\sqrt{15x} + 3\sqrt{6}}{3x}}$$

Ex③: $\dfrac{6\sqrt{5} - 2\sqrt{24x^3}}{2\sqrt{x}}$, $x > 0,\ x \in \mathbb{R}$

$$\frac{6\sqrt{5} - 2\sqrt{4 \cdot 6x^2 \cdot x}}{2\sqrt{x}}$$

$$\frac{\overset{3}{\cancel{6}}\sqrt{5} - \overset{2}{\cancel{4}}x\sqrt{6x}}{\cancel{2}\sqrt{x}}$$

$$\left(\frac{3\sqrt{5} - 2x\sqrt{6x}}{(\sqrt{x})}\right)\left(\frac{\sqrt{x}}{(\sqrt{x})}\right)$$

$$= \frac{3\sqrt{5x} - 2x\sqrt{6x^2}}{\sqrt{x^2}} = \frac{3\sqrt{5x} - 2x \cdot x\sqrt{6}}{x}$$

$$= \frac{3\sqrt{5x} - 2x^2\sqrt{6}}{x}$$