UNIT 3: Rational Functions

Rational Function:

$$P(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

for us: $f(x)$ and $g(x)$ will be polynomials restricted to monomials, binomials and trinomials.

Graphing and analyzing Rational functions paying particular attention to asymptotes, points of discontinuity, intercepts and domain and range.

Ex: Analyze $f(x) = \dfrac{x^2 - 3x - 4}{x^2 - 1}$

Step 1: factor num. and den.

$$f(x) = \frac{(x-4)(x+1)}{(x-1)(x+1)}$$

NPV $\Rightarrow x \neq \pm 1$

Roots of num $\Rightarrow x = -1$, $x = 4$

Note: if a root is common to both the numerator and denominator we get a pt. of discontinuity (hole) at that value of x.

So in this example the P. of D. is at $x = -1$ (and $y = \frac{5}{2}$)

Note 2: The other NPV's (Roots of denominator that are not roots of numerator) will give vertical asymptotes.

So here, the V.A. is at $x = 1$

Note 3: The other roots of the numerator (that were not also roots of the denominator) will be x-intercepts

i.e. $\dfrac{(x-4)(x+1)}{(x-1)(x+1)} = 0 \Rightarrow (x-4)(x+1) = 0$

$x = 4$ or $\boxed{x = -1}$ NPV

So x-int is $x = 4$

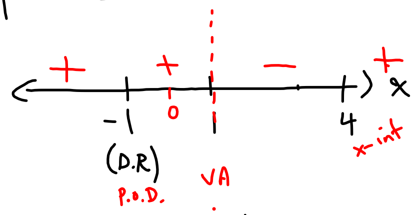
Step 2 Get y-intercept ⇒ evaluate $f(0)$
(Best to do in original function.)

$$f(0) = \frac{0^2 - 3(0) - 4}{0^2 - 1} = 4$$

y-int
(0,4)

Step 3: put All roots on a number line

Test x=0
see y-intercept



and test some x-values to determine location of function w.r.t. the x-axis
i.e. are the y-values pos. or neg.

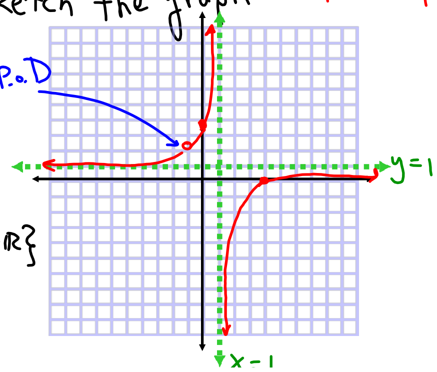
Step 4: Determining horizontal (H.A) or oblique (O.A.) asymptote

H.A ⇒ when num and den. are same degree, reduce the coefficients of the degree terms (leading coefficient)

$$f(x) = \frac{1x^2 - 3x - 4}{1x^2 - 1} \Rightarrow \text{H.A. } y = \frac{1}{1} = 1$$

NOTE: O.A. ⇒ when num is one degree higher than den. use long division and $\boxed{y = \text{quotient}}$ gives the asymptote equation.

Step 5: Sketch the graph



D: {x | x ≠ ±1, x ∈ ℝ}

R: {y | y ≠ 1, y ≠ $\frac{5}{2}$, y ∈ ℝ}

Ex.2 Analyze: $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 5x - 6}$

y-int ⇒ $f(0) = \dfrac{6}{-6} = -1$

factor $f(x) = \dfrac{(x-3)(x-2)}{(x-6)(x+1)}$ $x = 3, x = 2$

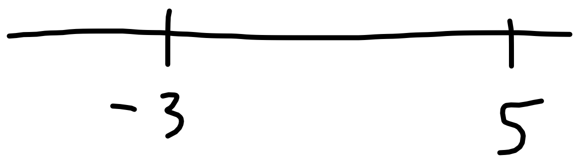
$x = 6$ $x = -1$

x-int are $x = 3$ $x = 2$

V.A. are $x = 6$ $x = -1$

H.A. is $y = 1$

$$\frac{-3+5}{2}$$



$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \qquad x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\frac{-2b}{2a} = \frac{-b}{a} \div 2$$

$$= \frac{-b}{a} \times \frac{1}{2} = \frac{-b}{2a}$$

(iii) $y = \dfrac{x-1}{x^2-x-6}$

y-int: $\boxed{x=0}$ $\dfrac{0-1}{0^2-0-6} = \dfrac{1}{6}$

$y = \dfrac{x-1}{(x-3)(x+2)}$

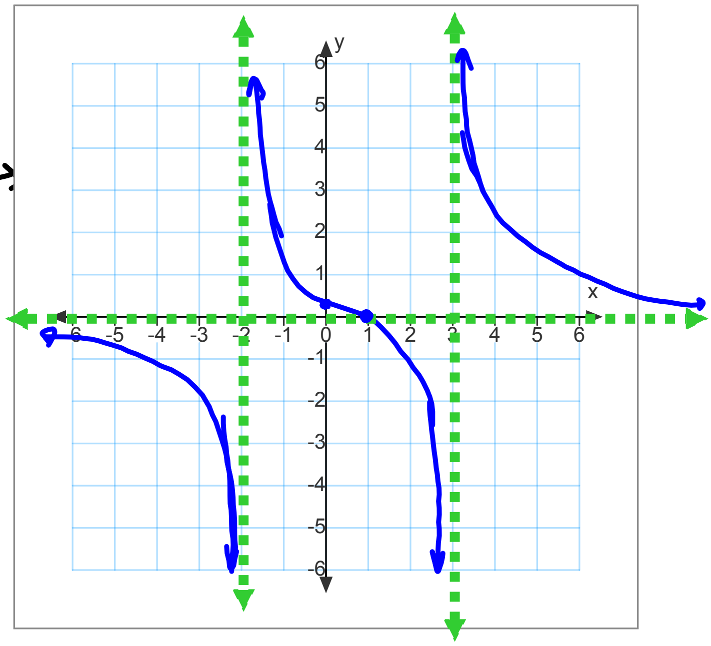
zero of num: $x = 1$

zeros of den: $x = 3$, $x = -2$

Since num. is lesser degree than den., the H.A. is $y=0$

x-int ⇒ $x=1$

V.A. $x=3$, $x=-2$



Ex: $y = \dfrac{x^2 - 3x - 4}{x + 2}$

oblique Asymptote

$y = \dfrac{(x-4)(x+1)}{(x+2)}$

y-int $(0,-2)$

x-int $(4,0)$
$(-1,0)$

NPV: $x \neq -2$

V.A. $x = -2$

$-$ $+$ $-$ $+$

-2 -1 4

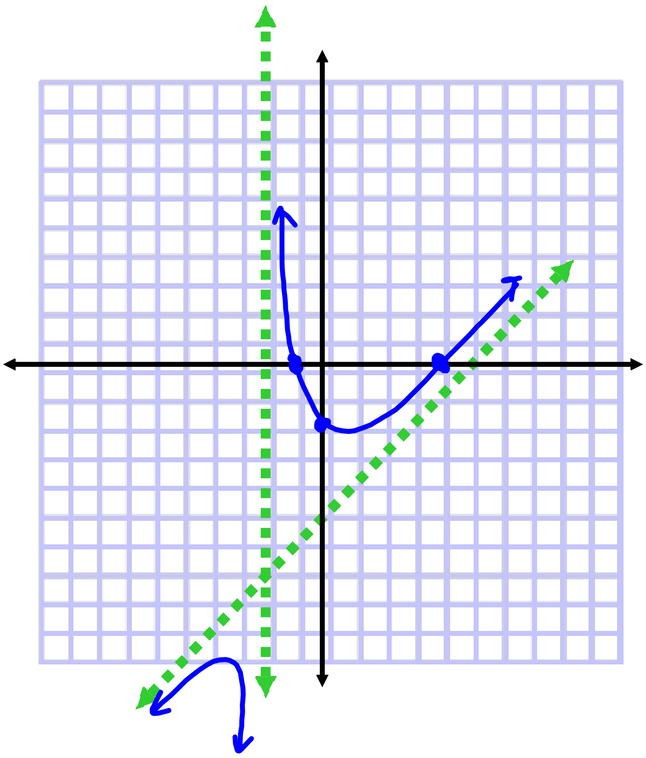
Oblique Asymptote:

$$
\begin{array}{r}
x - 5 \\
x+2 \overline{\smash{\big)}\, x^2 - 3x - 4} \\
-(x^2 + 2x) \\
\hline
-5x - 4 \\
-(-5x - 10) \\
\hline
\end{array}
$$

y = quotient is O.A.

so, $y = x - 5$

is oblique Asymptote



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