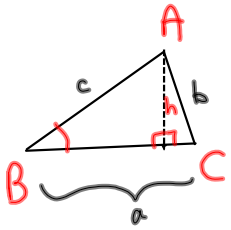
3.1 Exploring side-angle relationships in Acute Triangles



$\sin B = \frac{h}{c}$

$h = c \sin B$

Also, $\sin C = \frac{h}{b}$

$h = b \sin C$

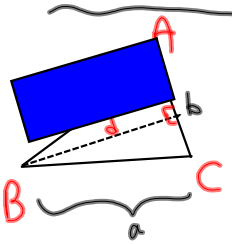
Since $h = c \sin B$ and $h = b \sin C$

then $\frac{c \sin B}{\sin C} = \frac{b \sin C}{\sin C}$

$\frac{c \sin B}{\sin C} = b$

$\frac{c \sin B}{\sin C \sin B} = \frac{b}{\sin B}$

Therefore $\frac{c}{\sin C} = \frac{b}{\sin B}$



$\frac{\sin A}{1} = \frac{d}{c}$

$d = c \sin A$

$\sin C = \frac{d}{a}$

$d = a \sin C$

Since $d = c \sin A$ and $d = a \sin C$

then $\frac{c \sin A}{\sin A \sin C} = \frac{a \sin C}{\sin A \sin C}$

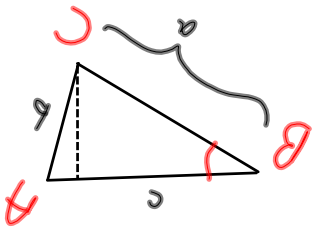
$\frac{c}{\sin C} = \frac{a}{\sin A}$

and since $\frac{c}{\sin C} = \frac{b}{\sin B}$

we can finally say that in an acute ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This says, for any acute Δ the ratio of a side with the sin of its opposite angle will be equal for all 3 sides and their corresponding opposite angles.



$\sin B = \frac{h}{c}$

$h = c \sin B$

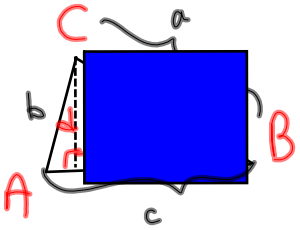
Also $\frac{\sin C}{1} = \frac{h}{b}$

$h = b \sin C$

Since $h = c\sin B$ and $h = b\sin C$ we can say that $c\sin B = b\sin C$

$\frac{c \sin B}{\sin B \sin C} = \frac{b \sin C}{\sin B \sin C}$

$\frac{c}{\sin C} = \frac{b}{\sin B}$



Same Δ as before rotated to use different base and height.

$\sin B = \frac{d}{a}$

$d = a \sin B$

$\sin A = \frac{d}{b}$

$d = b \sin A$

since $d = b\sin A$ and $d = a \sin B$ then $b\sin A = a \sin B$

now divide both sides by $\sin A \sin B$

$\frac{b \sin A}{\sin A \sin B} = \frac{a \sin B}{\sin A \sin B}$

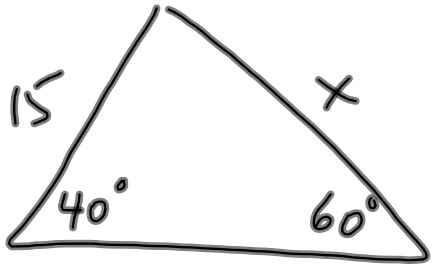
so we get $\frac{b}{\sin B} = \frac{a}{\sin A}$

from earlier we had $\frac{c}{\sin C} = \frac{b}{\sin B}$

∴ we can say that

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Ex: Using this relationship:

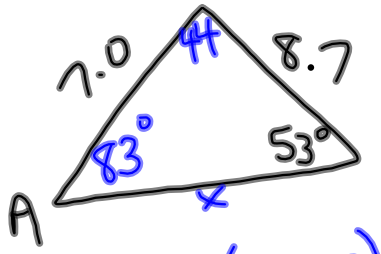


$$\frac{X}{\sin 40^\circ} = \frac{15}{\sin 60^\circ} \cdot \sin 40$$

$$X = \frac{15 \sin 40^\circ}{\sin 60^\circ}$$

$$X = 11.1$$

Ex ②: $\frac{7.0}{\sin 53^\circ} = \frac{8.7}{\sin A}$



$180 - (83 + 53)$

$180 - 136$

44°

$$\frac{7.0 \sin A}{7.0} = \frac{8.7 \sin 53^\circ}{7.0}$$

$$\sin A = \frac{8.7 \sin 53^\circ}{7.0}$$

$$\sin A = 0.9926$$

$$A = \sin^{-1}(0.9926)$$

$$A = 83^\circ$$

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2. **i)** Sketch a triangle that corresponds to each equation below.
ii) Solve for the unknown side length or angle measure. Round your answer to one decimal place.

a) $\frac{w}{\sin 50°} = \frac{8.0}{\sin 60°}$ 7.1

b) $\frac{k}{\sin 43°} = \frac{9.5}{\sin 85°}$ 6.5

c) $\frac{6.0}{\sin M} = \frac{10.0}{\sin 72°}$ 35°

d) $\frac{12.5}{\sin Y} = \frac{14.0}{\sin 88°}$ 63°