MATH 3208
COMPOSITE FUNCTIONS

Two functions, f(x) and g(x), can be combined using composition to produce two new functions, f(g(x)) and g(f(x)).

DIFFERENT REPRESENTATIONS OF COMPOSITE FUNCTIONS

The composite function $f(g(x))$ can be written as $(f \circ g)(x)$. $f(g(x))$
The composite function $g(f(x))$ can be written as $(g \circ f)(x)$.
The composite function $f(g(h(x)))$ can be written as $(f \circ g \circ h)(x)$.

EXAMPLE 1

Given the functions $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$.

(a) Evaluate each of the following:

(i) $g(h(-3))$

$h(-3) = (-3)^2 - 1 = 8$

$g(h(-3)) = g(8)$

$g(8) = 8+4 = 12$

(ii) $(h \circ f)\left(\frac{1}{2}\right) = h(f(\frac{1}{2}))$

$f(\frac{1}{2}) = 3 \cdot \frac{1}{2} = \frac{3}{2}$

$h(\frac{3}{2}) = (\frac{3}{2})^2 - 1 = \frac{5}{4}$

(iii) $f(g(h(\sqrt{5}))) = 24$

$h(\sqrt{5}) = (\sqrt{5})^2 - 1 = 4$

$g(4) = 4+4 = 8$

$f(8) = 3(8) = 24$

$f(g(h(\sqrt{5})))$

(b) Determine each of the following composite functions.

(i) $(g \circ f)(x)$

$g(f(x))$

$g(3x) = 3x + 4$

(ii) $f(h(x))$

$f(x^2-1)$

$= 3(x^2-1)$

$= 3x^2 - 3$

(iii) $(h \circ g)(x)$

$h(g(x))$

$h(x+4) =$

$(x+4)^2 - 1$

(iv) $f(g(h(x)))$

$f(g(x^2-1))$

$g(x^2-1) = (x^2-1) + 4$

$g(x^2-1) = x^2 - 1 + 4$

$g(x^2-1) = x^2 + 3$

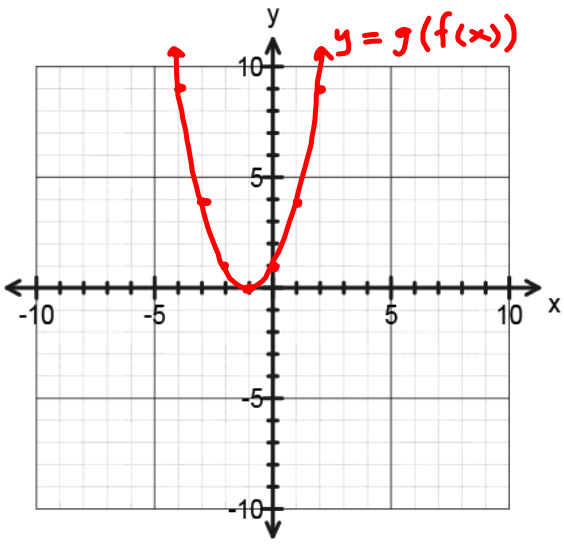
$f(x^2+3) = 3(x^2+3)$

$f(g(h(x))) = 3x^2 + 9$

EXAMPLE 2

Given the functions $f(x) = x + 1$ and $g(x) = x^2$.

Determine the equation of the composite function $y = g(f(x))$, graph the composite function, and state its domain and range.



$y = g(x+1)$

$y = (x+1)^2$

vertex $(-1, 0)$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

EXAMPLE 3

Given the functions $f(x) = x^2 + 5x + 1$, $g(x) = 2x + 1$, $h(x) = \sqrt{x - 2}$, and $k(x) = |x + 5|$.

Determine the following composite functions and state their domain and range.

(a) $g(f(x))$ (b) $f(h(x))$ (c) $h(g(x))$ (d) $k(f(x))$

EXAMPLE 4

If $h(x) = f(g(x))$, determine f(x) and g(x) for each of the following:

(a) $h(x) = \sqrt{x^3 + 1}$

$g(x) = x^3$

$f(x) = \sqrt{x+1}$

(b) $h(x) = (x-2)^2 + (x-2) + 3$

$g(x) = x - 2$

$f(x) = x^2 + x + 3$

EXAMPLE 5

A spherical weather balloon is being inflated. The balloon's radius, r, in feet, after t minutes is given by $r = \sqrt{t}$.

(a) Express the surface area of the balloon as a function of time, t. $(Area = 4\pi r^2)$

$Area = 4\pi(\sqrt{t})^2$

$Area = 4\pi t$

(b) After how many minutes will the surface area be 180 ft^2?

$180 = 4\pi t$

$\frac{180}{4\pi} = t$

$t = \frac{45}{\pi}$

$t \doteq 14.3$ minutes

(c) Express the volume of the balloon as a function of time, t. $\left(Volume = \frac{4}{3}\pi r^3\right)$

$V = \frac{4}{3}\pi(\sqrt{t})^3$

$V = \frac{4}{3}\pi t^{\frac{3}{2}}$

$V = \frac{4}{3}\pi\sqrt{t^3}$

$V = \frac{4}{3}\pi t\sqrt{t}$

EXAMPLE 6

Given $f(x) = \frac{1}{x+1}$ and $g(x) = \frac{1}{x}$, determine the functions f(g(x)) and f(f(x)).

$$f(g(x)) = \frac{1}{\frac{1}{x}+1} \cdot \frac{x}{x} = \frac{x}{1+x}$$

$$f(f(x)) = \frac{1}{\frac{1}{x+1}+1} \cdot \frac{x+1}{x+1} = \frac{x+1}{1+x+1} = \frac{x+1}{x+2}$$

QUESTIONS Pages 507 – 509, # 1a, d, 2a, c, 3a, c, 4e, f, 5a, b, 6, 7, 14, 16a, 17, 18a, 22