3.2 The Remainder Theorem

Remember long division:

$120 \div 9$

$$\frac{120}{9} = \overset{Q}{13} + \frac{3}{9}\ \frac{R}{D}$$

$$\begin{array}{r} 13 \ Q \\ D\ 9\,\big)\,\overline{120} \\ -\ 9 \\ \hline 30 \\ -27 \\ \hline 3 \ R \end{array}$$

Long Division for polynomials

Ex: $x^3 + 3x^2 - 2x + 1 \div (x-2)$

$$\begin{array}{r} x^2 + 5x + 8 \\ x - 2\,\big)\,\overline{x^3 + 3x^2 - 2x + 1} \\ -(x^3 - 2x^2) \qquad\qquad \\ \hline 5x^2 - 2x \qquad \\ -(5x^2 - 10x) \qquad \\ \hline 8x + 1 \\ -(8x - 16) \\ \hline \boxed{17} \end{array}$$

17 Remainder

Note: make sure dividend is in descending order with no terms missing.

This means we can write

$$\frac{x^3 + 3x^2 - 2x + 1}{x - 2} = x^2 + 5x + 8 + \frac{17}{x-2}$$

$x - a$

Ex: $x^3 + 3x^2 - 2x + 1 \div x - 2$

$$
\begin{array}{r}
x^2 + 5x + 8 \quad \text{Quotient} \\
x - 2 \overline{\big) \cancel{x^3} + 3x^2 - 2x + 1} \\
-(\cancel{x^3} - 2x^2) \\
\hline
\cancel{5x^2} - 2x \\
-(\cancel{5x^2} - 10x) \\
\hline
\cancel{8x} + 1 \\
-(\cancel{8x} - 16) \\
\hline
\boxed{17} \quad \text{Remainder}
\end{array}
$$

$$\frac{x^3 + 3x^2 - 2x + 1}{x - 2} = x^2 + 5x + 8 + \frac{17}{x - 2}$$

Ex: use long division to determine the quotient and remainder when

$$2x^4 - 5x^3 + 4x^2 + 7x - 10 \div x+1$$

$Q = 2x^3 - 7x^2 + 11x - 4$ $R = -6$

Ex: $x^3 - 3x^2 - 4x + 12 \div x - 2$

$Q = x^2 - x - 6$ $R = 0$

use <u>synthetic division</u>:

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ & & 2 & -2 & -12 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

Remainder

quotient $\boxed{x^2 - x - 6}$

use synthetic division:

$x - (-1)$

$2x^4 - 5x^3 + 4x^2 + 7x - 10 \div x + 1$

need the "a" from $x - a$

in this example $a = -1$ (divisor)

"a"

coefficients

$$\begin{array}{r|rrrrr} -1 & 2 & -5 & 4 & 7 & -10 \\ & & -2 & 7 & -11 & 4 \\ \hline & 2 & -7 & 11 & -4 & \boxed{-6} \end{array}$$

multiply

Quotient is $2x^3 - 7x^2 + 11x - 4$

and Remainder is -6

The Remainder Theorem

When a polynomial, $P(x)$ is divided by $(x-a)$ the remainder is $P(a)$

page 124-125
1, 3, 4, 6, 7, 8, 9, 10, 14

3.3 The Factor Theorem

→ $(x-a)$ is a factor of polynomial, $P(x)$ if and only if $P(a)=0$

Integral Roots (Zero) Theorem

If $(x-a)$ is a factor of a polynomial $P(x)$, with integral coefficients then "a" is a factor of the constant term.

using Integral Roots Theorem to factor $P(x)$

Ex: Factor completely:

$$x^4 + 2x^3 + 2x^2 - 2x - 3$$

List of possible integral Roots : $\pm\{1, 3\}$

Test $x = 1$
$(x-1)$

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & 2 & -2 & -3 \\ & & 1 & 3 & 5 & 3 \\ \hline & 1 & 3 & 5 & 3 & \boxed{0} \end{array}$$

$R = 0$

$\therefore (x-1)$ is a factor

$\therefore x^4 + 2x^3 + 2x^2 - 2x - 3$

$(x-1)(x^3 + 3x^2 + 5x + 3)$ → Note: No need to test positives since all coefficients are pos.

Now test $x = -1$ in $x^3 + 3x^2 + 5x + 3$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 5 & 3 \\ & & -1 & -2 & -3 \\ \hline & \underbrace{1 \quad 2 \quad 3}_{\text{Quotient}} & & & \boxed{0} \end{array}$$

→ 0 Remainder $\therefore (x+1)$ is a factor

So now $x^4 + 2x^3 + 2x^2 - 2x - 3$ becomes

$$(x-1)(x+1)(x^2 + 2x + 3)$$

which finally becomes: $(x-1)(x+1)(x^2 + 2x + 3)$

since $x^2 + 2x + 3$ does not factor!

Ex: Factor completely:

p. 133 #1 - 7

$$x^4 - 5x^3 + 2x^2 + 20x - 24$$

$\pm\{1, 2, 3, 4, 6, 8, 12, 24\}$

$$(x-2)(x-2)(x+2)(x-3)$$

$$(x-2)^2(x+2)(x-3)$$

#7 (a) $x^2 - x + k$, $(x-2)$

If $(x-2)$ is a factor of $P(x) = x^2 - x + k$

then $P(2) = 0$

so we get $(2)^2 - (2) + k = 0$

$$2 + k = 0$$

$$k = -2$$