

3.4 Equations and graphs of polynomialsTerminology:

Should only use the term "roots" in reference to the solutions of an equation.

The term "zeros" is used in reference to the values of x that make a function equal zero.

The term "x-intercept" is used in reference to the value of x for which a graph touches or crosses the x -axis.

Ex: Determine the roots of

$$x(x-3)(x-2)(x+1) = 0$$

from factored form we apply the zero product property

Roots

$$\boxed{x=0} \quad x-3=0 \quad x-2=0 \quad x+1=0$$

$$\boxed{x=3} \quad \boxed{x=2} \quad \boxed{x=-1}$$

Ex 2: Determine the zeros of

$$f(x) = x^3 - x^2 - 4x + 4$$

set $f(x) = 0$ and factor

$$x^3 - x^2 - 4x + 4 = 0$$

$$x^2(x-1) - 4(x-1) = 0$$

$$(x-1)(x^2-4) = 0$$

$$(x-1)(x-2)(x+2) = 0$$

$$x-1=0 \quad x-2=0 \quad x+2=0$$

Zeros

$$\boxed{x=1} \quad \boxed{x=2} \quad \boxed{x=-2}$$

the graph of $f(x) = x^3 - 4x^2 + x + 6$

set $f(x) = 0$ and factor

$$x^3 - 4x^2 + x + 6 = 0$$

$$\text{P.I.R. } \pm \{1, 2, 3, 6\}$$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$\therefore x = -1$ is an
x-intercept

b/c $x+1$ is a factor

$$(x+1)(x^2 - 5x + 6) = 0$$

now solve $x^2 - 5x + 6 = 0$

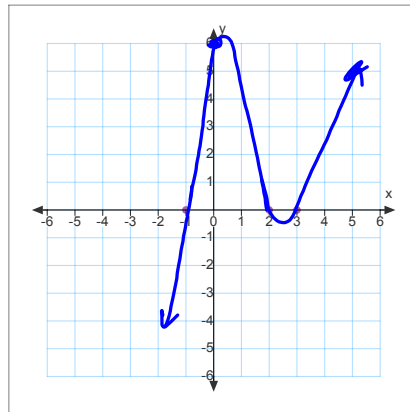
$$(x-2)(x-3) = 0$$

$$x-2=0 \quad x-3=0$$

$$x=2 \quad x=3$$

So, the x-intercepts are

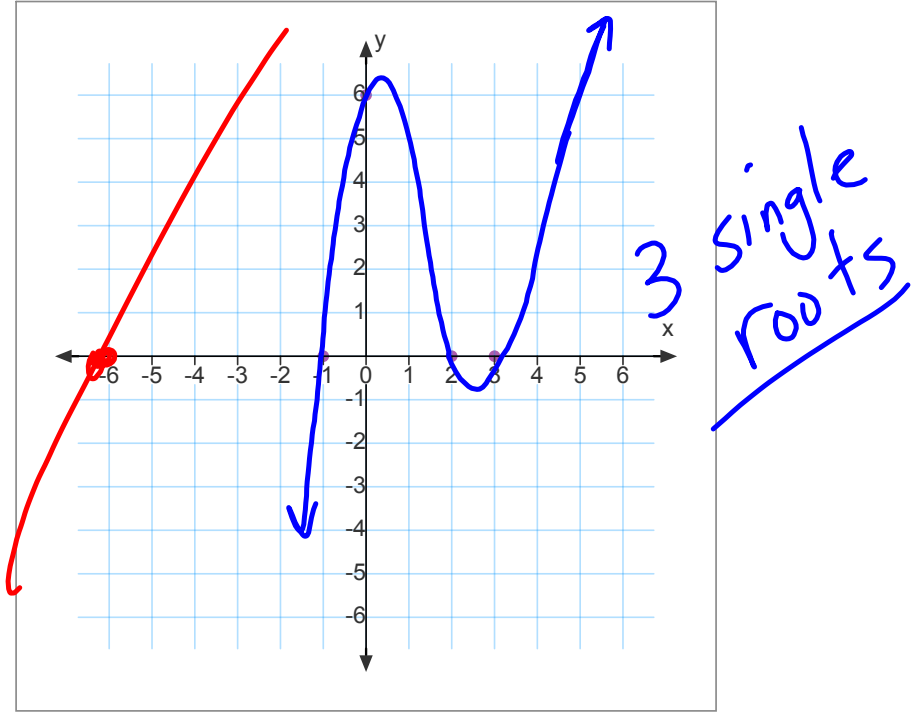
$$x = -1, x = 2, x = 3$$



Ex: Determine the zeros of .

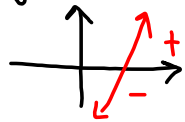
$$f(x) = x^3 - 12x^2 + 47x - 36$$

$x = 1$ is only real zero



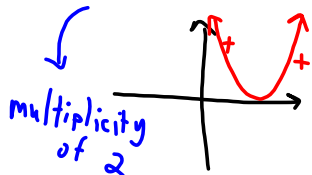
How do graphs behave at x-intercepts?

single "root" \Rightarrow graph passes directly through



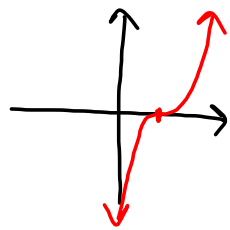
y-values change sign - to +
+ to -

double "root" \Rightarrow graph is tangent to x-axis



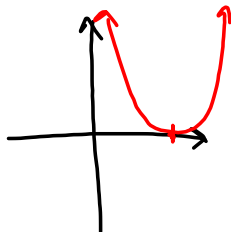
"bounces off"
y-values do not change
sign + to + or - to -

"triple root" \Rightarrow graph "flattens" ^{at the x-intercept} and then passes

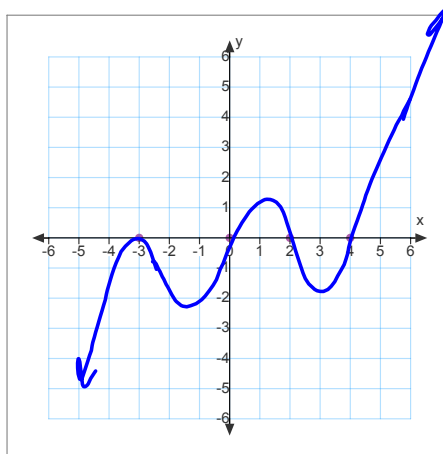


through x-axis. y-values do
change sign

quadruple "root" \Rightarrow flattens and bounces off
at the x-intercept

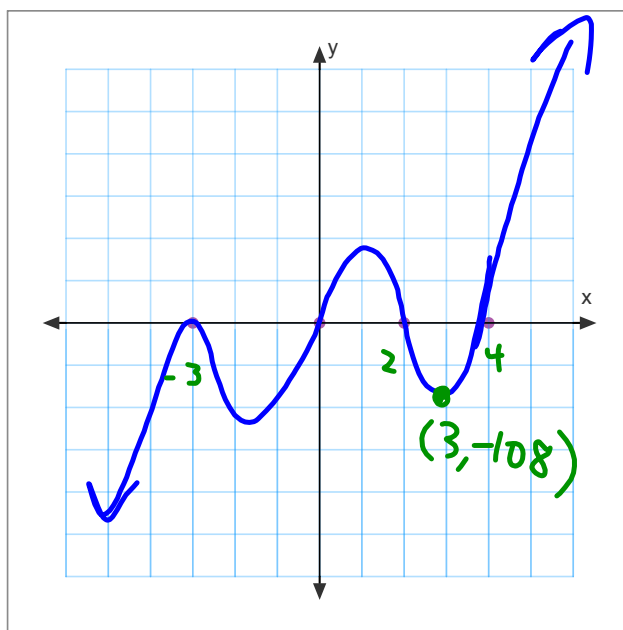


Ex: sketch $y = x(x-2)(x-4)(x+3)^2$



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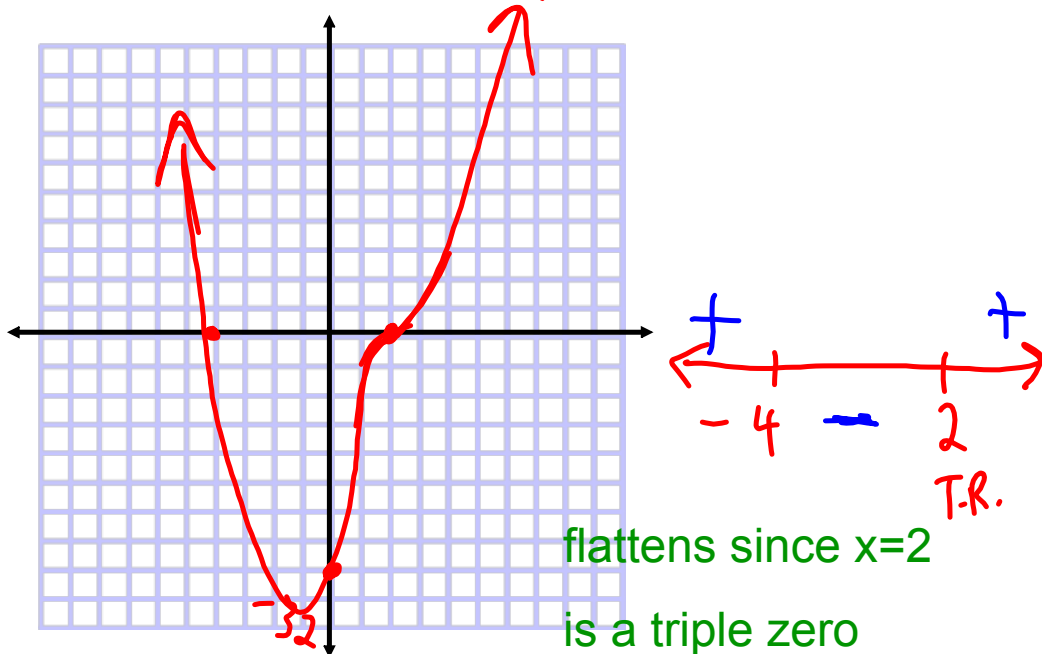
x-int: $x=0, x=2, x=4, x=-3$ (D.R.)



Ex sketch $y = (x - 2)^3(x + 4)$

x-int : $x = 2$ (triple)
 $x = -4$

y-int : $y = (0 - 2)^3(0 + 4) = -32$
 $(-8)(4)$



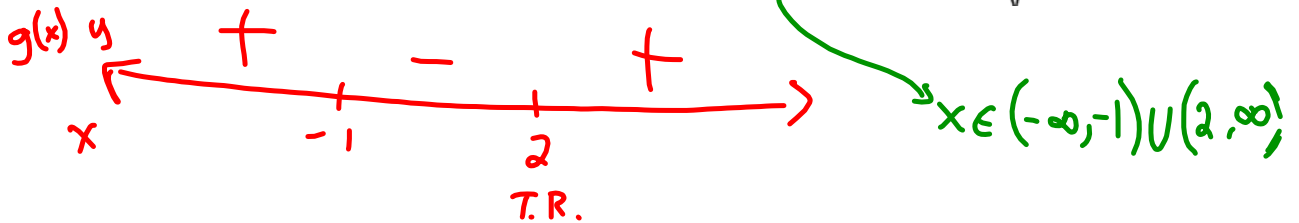
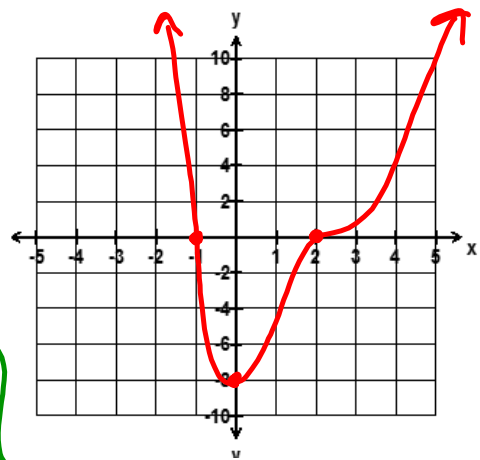
Ex: Sketch $y = (x - 1)(x - 3)^2(x + 4)^2$

Practice p. 147
 # 1, 2, 5, 7, 8, 9

Sketch a graph of each polynomial function without using technology. Complete the table by stating the characteristics of the polynomial functions to help sketch the graphs.

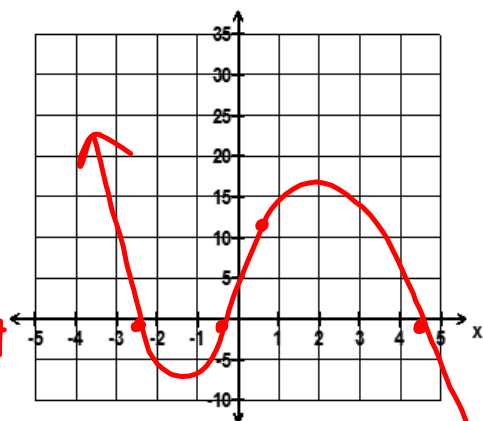
a. $g(x) = (x - 2)^3(x + 1)$

Degree	4
Leading coefficient	1
End Behaviour	Q2 to Q1
Zeros/x-intercepts	2 and -1
y-intercept	-8
Intervals of positive $g(x)$	$x < -1$ or $x > 2$
Intervals of negative $g(x)$	$-1 < x < 2$



b. $f(x) = -x^3 + 13x + 12$

Degree	3
Leading coefficient	-1
End Behaviour	Q2 to Q4
Zeros/x-intercepts	-3, -1, 4
y-intercept	(0, 12)
Intervals of positive $f(x)$	$x < -3$ or $-1 < x < 4$
Intervals of negative $f(x)$	$-3 < x < -1$ or $x > 4$



x-int: $-x^3 + 13x + 12 = 0 \rightarrow x^3 - 13x - 12 = 0$

P.I.R $\pm \{1, 2, 3, 4, 6, 12\}$

$$\begin{array}{r} -1 \overline{) 1 \ 0 \ -13 \ -12} \\ \underline{-1 \ 1 \ 12} \\ 1 \ -1 \ -12 \ 0 \end{array}$$

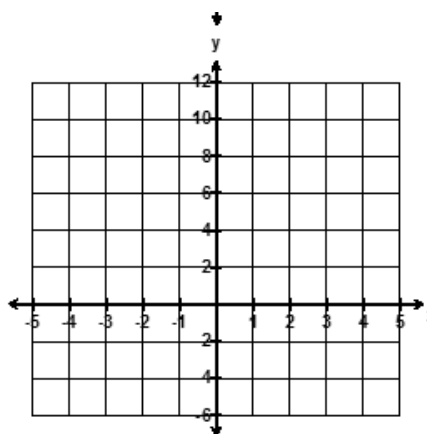
$(x+1)(x^2 - x - 12)$

$(x+1)(x-4)(x+3)$

$x = -1 \quad x = 4 \quad x = -3$

c. $h(x) = (x - 1)^2(x + 2)^3$

Degree	
Leading coefficient	
End Behaviour	
Zeros/x-intercepts	
y-intercept	
Intervals of positive $h(x)$	
Intervals of negative $h(x)$	



Domain: {

Range: {

Factor sum/difference of cubes

$$x^3 - 1$$

$$(x-1)(x^2 + x + 1)$$

$$x^3 + 8$$

$$(x+2)(x^2 - 2x + 4)$$

$$x^3 - 27$$

$$(x-3)(x^2 + 3x + 9)$$

$$x^3 - 125$$

$$(x-5)(x^2 + 5x + 25)$$

$$\square^3 - \triangle^3$$

$$(\square - \triangle)(\square^2 + \square\triangle + \triangle^2)$$

$$\text{house}^3 + \text{parallelogram}^3$$

$$(\text{house} + \text{parallelogram})(\text{house}^2 - \text{house}\text{parallelogram} + \text{parallelogram}^2)$$

Problem solving with Polynomials

Ex: The product of 4 consecutive odd integers is 945, what are the integers?

Solution: Let x be 1st # 3
 then $x+2$ is 2nd 5
 $x+4$ is 3rd 7
 and $x+6$ is 4th 9

Eqⁿ is $x(x+2)(x+4)(x+6) = 945$
 $x(x+2)(x^2 + 10x + 24) = 945$
 $x(x^3 + 10x^2 + 24x + 2x^2 + 20x + 48) = 945$
 $x(x^3 + 12x^2 + 44x + 48) = 945$
 $x^4 + 12x^3 + 44x^2 + 48x - 945 = 0$
 $\pm 1, 3, 5, 7, 9, \dots$

$3 \mid 1 \ 12 \ 44 \ 48 \ -945$
 $\quad 3 \ 45 \ 267 \ 945$
 $\hline 1 \ 15 \ 89 \ 315 \ 0$ $x=3$ is a root

$-3 \mid 1 \ 15 \ 89 \ 315$
 $\quad -3 \ -36 \ -159$
 $\hline 1 \ 12 \ 53 \ \times$

$-5 \mid 1 \ 15 \ 89 \ 315$
 $\quad -5 \ -50 \ -195$
 $\hline 1 \ 10 \ 39 \ \times$

$-7 \mid 1 \ 15 \ 89 \ 315$
 $\quad -7 \ -56 \ -231$
 $\hline 1 \ 8 \ 33 \ \times$

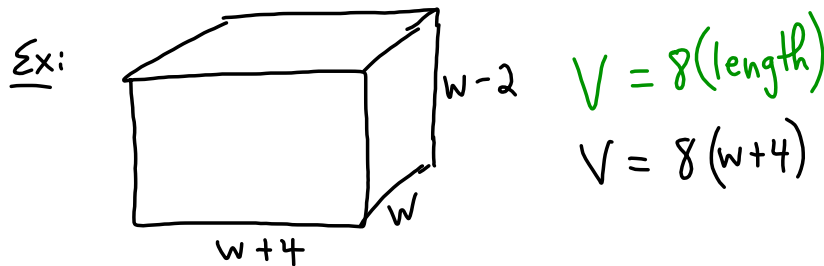
$-9 \mid 1 \ 15 \ 89 \ 315$
 $\quad -9 \ -54 \ -315$
 $\hline 1 \ 6 \ 35 \ 0$
 $\underbrace{\hspace{2cm}}_{x^2 + 6x + 35}$

so $x=3$ or $x=-9$

If $x=3$ then numbers are 3, 5, 7, and 9

If $x=-9$ then " " -9, -7, -5, -3

The width of a rectangular prism is w centimetres. The height is 2 cm less than the width. The length is 4 cm more than the width. If the magnitude of the volume of the prism is 8 times the measure of the length, what are the dimensions of the prism?



$$l \times w \times h = V$$

$$(w+4)(w)(w-2) = 8(w+4)$$

$$w(w+4)(w-2) = 8w+32$$

$$w(w^2+2w-8) = 8w+32$$

$$w^3+2w^2-8w-8w-32=0$$

$$w^3+2w^2-16w-32=0$$

$$w^2(w+2)-16(w+2)=0$$

$$(w^2-16)(w+2)=0$$

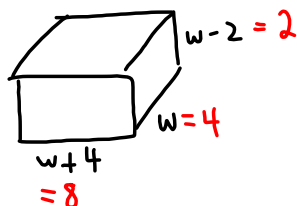
$$(w+4)(w-4)(w+2)=0$$

$$\cancel{w=-4} \quad \boxed{w=4} \quad \cancel{w=-2}$$

P 150
 $\neq 12, 13, 16, 15$


$$V = 64$$

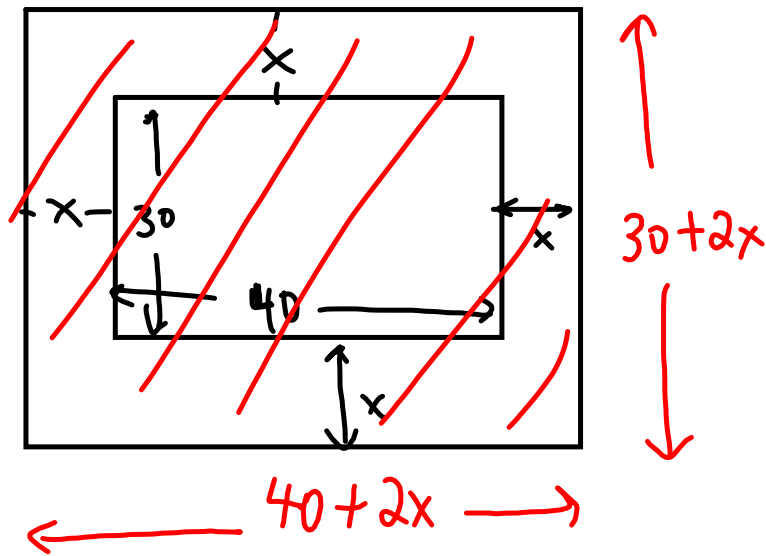
cannot have neg. length



Dimensions are 2, 4 and 8

13

 = 2000 m²



$$(40 + 2x)(30 + 2x) = 2000$$

$$1200 + 80x + 60x + 4x^2 = 2000$$

$$\frac{4x^2 + 140x - 800 = 0}{4}$$

$$x^2 + 35x - 200 = 0$$

$$(x + 40)(x - 5) = 0$$

~~$$x = -40 \text{ OR } x = 5$$~~

Inadmissible Boardwalk is 5 m wide

Writing equations from roots
and/or functions from zeros/x-intercepts
(graphs)

Ex: Determine an equation that would
generate the roots $x=2, x=-3, x=5$

Solution:

$$\begin{array}{ccc} x=2 & x=-3 & x=5 \\ x-2=0 & x+3=0 & x-5=0 \\ (x-2)(x+3)(x-5)=0 \end{array}$$

Ex: Determine a function that would
generate the following graph

x-int are

$$x = -2$$

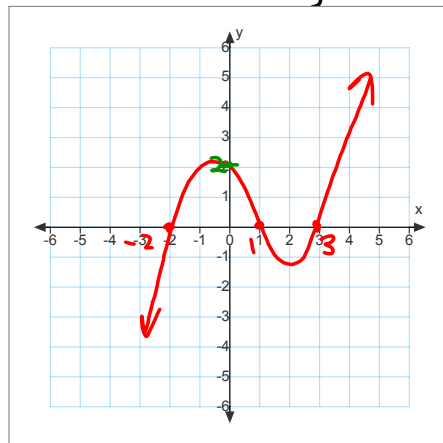
$$x = 1$$

$$x = 3$$

y-int: $y = 2$

$$(0, 2)$$

x, y



$$y = a(x+2)(x-1)(x-3)$$

$$2 = a(0+2)(0-1)(0-3)$$

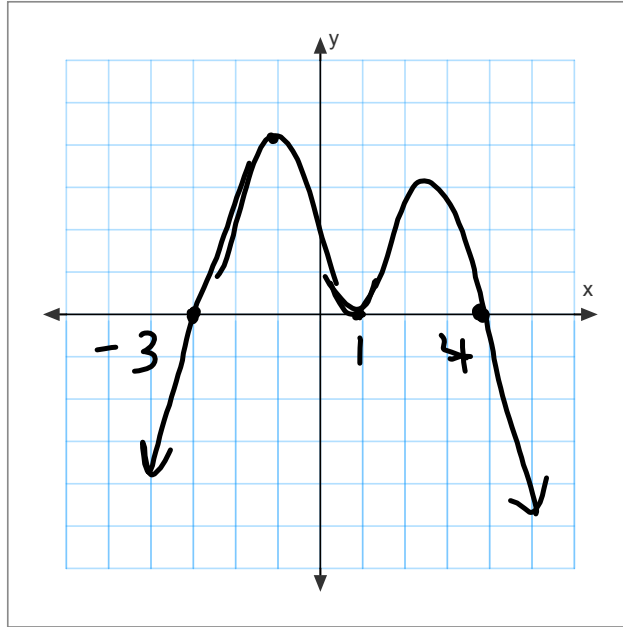
$$2 = a(2)(-1)(-3)$$

$$2 = 6a$$

$$\frac{1}{3} = a$$

$$\text{Function is } y = \frac{1}{3}(x+2)(x-1)(x-3)$$

x -int
 $x = -3$
 $x = 1$ (D.R.)
 $x = 4$



$$y = -(x+3)(x-1)(x-1)(x-4)$$

$$y = -(x+3)(x-1)^2(x-4)$$

P. 148 # 3, 4, 5

P. 149 # 10

P. 151 # 20