Mathematics 3201
Sample Mid-Year Exam #1, 2014-15
Item Breakdown/Solutions

PART I:

#	Ans	2M	2A	L3	Guide	Outcome
1	C	X			p.22	LR2
2	D	X			p.26	LR2
3	D	X			p.24	LR2
4	B		X		p.26	LR2
5	B		X		p.35	LR2
6	B			X	28/30	LR2
7	A		X		p.34	LR2
8	B	X			p.52	P4
9	B		X		p.52	P4
10	D	X			p.56	P4
11	C	X			p.52	P4
12	A	X			p.56	P4
13	A		X		p.60	P5
14	A		X		p.68	P5
15	A		X		p.70	P6
16	C		X		p.68	P5
17	A	X			p.72	P6
18	C		X		80/82	P1

#	Ans	2M	2A	L3	Guide	Outcome
19	C	X			p.92	P3
20	A	X			p.84	P6
21	B		X		88/90	P2
22	A		X		84	P5
23	C		X		84/94	P3/P6
24	C		X		p.80	P1
25	D			X	p.84	P6
26	B	X			p.100	RF1
27	C	X			p.100	RF1
28	D	X			p.102	RF1
29	D		X		p.104	RF1
30	C	X			p.104	RF1
31	D		X		p.106	RF2
32	B		X		p.106	RF2
33	B	X			p.108	RF2
34	D		X		p.108	RF2
35	A			X	p.110	RF3

PART II:

Item	Value	L2A	L3	Guide	Outcome
36a	3	X		p.34	LR2
36bi	1		X	p.36	LR2
36bii	2	X		p.36	LR2
37a	2	X		p.68	P5
37b	3	X		p.60	P5
37c	3		X	p.70	P6
37d	2		X	p.70	P6
38a	3	X		p.84	P5

Item	Value	L2A	L3	Guide	Outcome
38b	3	X		88/90	P2
38c	3		X	p.84	P6
39ai	1		X	p.106	RF2
39aii	3	X		p.106	RF2
39b	2	X		p.106	RF2
39c	4	X		p.114	RF3

PART II - Total Value: 35 marks

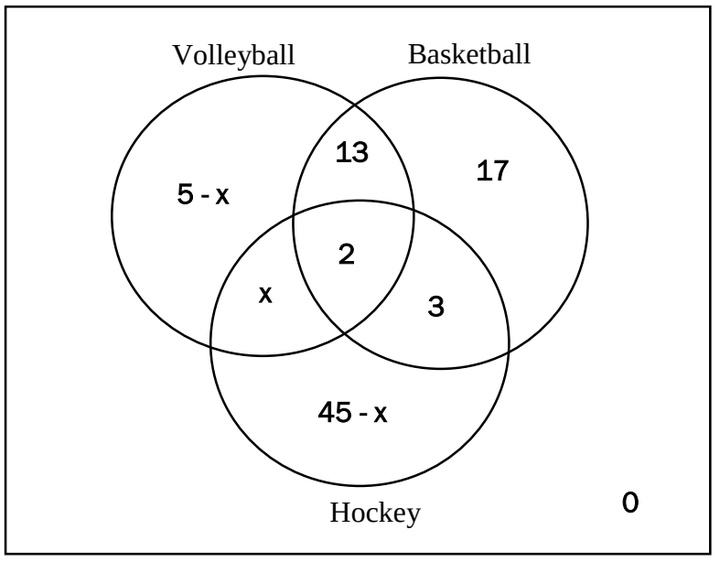
Answer **ALL** items in the space provided. Show **ALL** workings.

Value

3 36(a). There are 79 children who play sports at the recreation centre.

- 20 play volleyball
- 35 play basketball
- 50 play hockey
- 15 play only volleyball and basketball
- 5 play only basketball and hockey
- 2 play all three

Draw a Venn diagram to illustrate this information and use it to determine how many children just play hockey.



Venn Diagram (0.5 mark)

$(5 - x) + x + (45 - x) + 2 + 13 + 3 + 17 + 0 = 79$ (1 mark)

$85 - x = 79$ (0.5 mark)

$-x = -6$

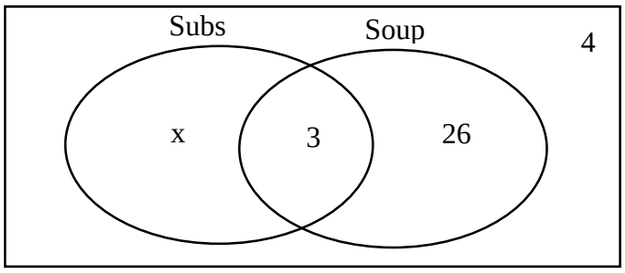
$x = 6$ (0.5 mark)

The # of children who play hockey only is 45 - 6 = 39 (0.5 mark)

Value

36(b). A student was asked the following question:
"40 people were surveyed. 26 people like soup, 3 people like both soup and submarine sandwiches, and 4 people like neither. How many people like submarine sandwhiches only?"

The student's solution to the question is shown below:

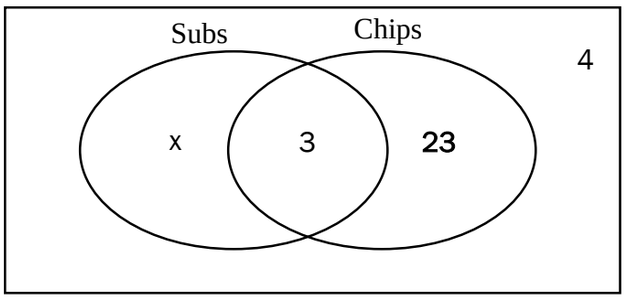


$x + 3 + 26 + 4 = 40$
$33 + x = 40$
$x = 7$; 7 people like submarine sandwiches only.

1 i) Identify the error in the student's solution.

The 26, should be 23. Forgot to account for overlap (1 mark)

2 ii) Algebraically determine the correct solution.



$x + 3 + 23 + 4 = 40$ (1 mark)
$30 + x = 40$ (0.5 mark)
$x = 10$ (0.5 mark)
10 people like Submarine sandwiches only.

2 37(a). How many different arrangements are there using the letters in the word **CRANBERRIES** ? Show your workings.

$\frac{11!}{3!\,2!}$ (0.5 mark)
(0.5 mark)

$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times \cancel{6} \times 5 \times 4 \times 3 \times \cancel{2} \times 1}{\cancel{6} \times \cancel{2}}$ (0.5 mark)

$= 3\,326\,400$
(0.5 mark)

Value

3 37(b). Algebraically solve for n: $_{n+2}C_2 = 21$

$$\frac{(n+2)!}{2![(n+2)-2]!} = 21$$

First two steps (1 mark)

$$\frac{(n+2)!}{2!\,n!} = 21$$

$$\frac{(n+2)(n+1)n!}{2 \times n!} = 21 \quad \text{(0.5 mark)}$$

$$n^2 + 3n + 2 = 42$$

$$n^2 + 3n - 40 = 0 \quad \text{(0.5 mark)}$$

$$(n+8)(n-5) = 0 \quad \text{(0.5 mark)}$$

~~$n = -8$~~ $or\ n = 5$ (0.5 mark)

3 37(c). There are 5 boys and 6 girls in a club. 5 club members will be selected for a field trip. How many ways can the teacher select 5 members if there must be at most 2 boys?

$(_5C_0)(_6C_5) + (_5C_1)(_6C_4) + (_5C_2)(_6C_3)$ (1.5 marks)

$= (1)(6) + (5)(15) + (10)(20)$ (0. 5 mark)

$= 6 + 75 + 200$ (0.5 mark)

$= 281$ (0.5 mark)

2 37(d). How many numbers greater than 400 are possible using the digits 2, 3, 4, and 5 if repetition is not allowed? Show your workings.

3 digits 4 digits

$(2)(3)(2) + (4)(3)(2)(1)$ (1.5 marks)

$= 12 + 24$

$= 36$ (0.5 mark)

3 38(a). Andrew cannot remember the correct order of the six digits in his ID number. He does remember that his number contains the digits 5, 6, 4, 3, 2, and 9. What is the probability that the first three digits of his number are all odd?

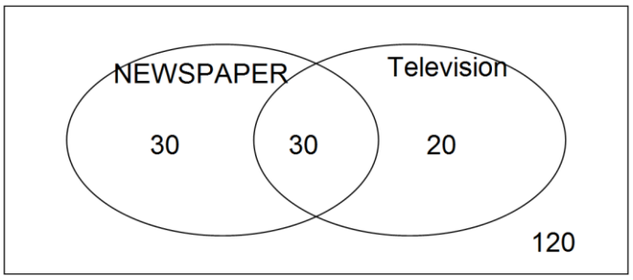
The first 3 digits of his number are odd and the last 3 are even. There are $_3P_3$ to arrange the first 3 odd digits (0.5 mark)and $_3P_3$ ways to arrange the last 3 even digits(0.5 mark). There are $_6P_6$ ways to arrange the 6 digits (0.5 mark). Therefore:

$$\text{P(first three digits are odd)} = \frac{_3P_3 \times {}_3P_3}{_6P_3} \quad \text{(1 mark)}$$

$$= \frac{6 \times 6}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{20} \quad \text{(0.5 mark)}$$

Value

3 38(b). A computer store advertised its annual half-price sale in the newspaper and on television. A survey of 200 customers indicated that 60 read about the sale in the newspaper, 50 watched the sale advertised on television, and 30 saw the sale through both sources. What is the probability that a randomly selected customer did not see the advertisement in either form?



$60 - 30 = 30$ *# of newspaper only* (0.5 mark)
$50 - 30 = 20$ *# of TV only* (0.5 mark)
Thus, 80 *who saw ad in at least one form* (0.5 *mark*)
$200 - 80 = 120$ *who did't see it in either form* (0.5 mark)
Therefore $P($ *did not see*$) = \frac{120}{200} = \frac{3}{5} = 0.6$ (1 mark)

OR $60 + 50 - 30 = 80$ *are the number of customers who saw it in at least one form* (1.5 *marks*)
$200 - 80 = 120$ *who did't see it in either form* (0.5 mark)
Therefore $P($ *did not see*$) = \frac{120}{200} = \frac{3}{5} = 0.6$ (1 mark)

3 38(c). There are 14 girls and 6 boys on the student council at a school. Five students from the student council are to be randomly selected to participate in a local town debate. What is the probability that at least 3 of the students selected are girls?

$$\frac{({}_{14}C_3 \times {}_6C_2)+({}_{14}C_4 \times {}_6C_1)+({}_{14}C_5 \times {}_6C_0)}{{}_{20}C_5} = \frac{\left(\frac{14!}{3!11!}\right)\left(\frac{6!}{2!4!}\right)+\left(\frac{14!}{4!10!}\right)\left(\frac{6!}{1!5!}\right)+\left(\frac{14!}{5!9!}\right)(1)}{\frac{20!}{5!15!}}$$ (2 marks)

$$= \frac{(364)(15)+(1001)(6)+(2002)(1)}{15504} = \frac{5460+6006+2002}{15504} = \frac{13468}{15504} \sim 0.869$$ (1 mark)

39(a). Arnold simplified an expression as follows:

$$\frac{2x}{x^2-4} + \frac{7}{2x-4}$$

$$= \frac{2x}{(x+2)(x-2)} + \frac{7}{2(x-2)}$$ Step 1
$$= \frac{\cancel{2}x}{(x+2)(x-2)} + \frac{7}{\cancel{2}(x-2)}$$ Step 2
$$= \frac{x}{(x+2)(x-2)} + \frac{7(x+2)}{(x-2)(x+2)}$$ Step 3
$$= \frac{x+7x+14}{(x+2)(x-2)}$$ Step 4
$$= \frac{8x+14}{(x+2)(x-2)}$$ Step 5

1 (i) Identify the step in which the error occurred and explain the mistake.

The error occurred in Step 2. He could not divide off the 2. (1 mark)

Value

3 (ii) Correct the errors and simplify.

$$= \frac{2x}{(x+2)(x-2)} + \frac{7}{2(x-2)}$$ (0.5 mark)

$$= \frac{2x(2)}{(x+2)(x-2)(2)} + \frac{7(x+2)}{2(x-2)(x+2)}$$ (1 mark)

$$= \frac{4x}{2(x+2)(x-2)} + \frac{7(x+2)}{2(x-2)(x+2)}$$ (0.5 mark)

$$= \frac{4x+7x+14}{2(x+2)(x-2)}$$ (0.5 mark)

$$= \frac{11x+14}{2(x+2)(x-2)}$$ (0.5 mark)

2 39(b). Simplify: $\frac{x+5}{7x+14} \div \frac{x^2-25}{10(x+2)}$, $x \neq -5,\ -2,\ 5$

$$= \frac{x+5}{7(x+2)} \div \frac{(x-5)(x+5)}{10(x+2)}$$ (1 mark)

$$= \frac{\cancel{x+5}}{7\cancel{(x+2)}} \cdot \frac{10\cancel{(x+2)}}{(x-5)\cancel{(x+5)}}$$ (0.5 mark)

$$= \frac{10}{7(x-5)}$$ (0.5 mark)

4 39(c). A school group is going on a field trip. The total cost is \$900. If 10 more students sign up, the price per student will decrease by \$15. If x represents the number of students and the situation is modelled by the equation $\frac{900}{x} - \frac{900}{x+10} = 15$, algebraically determine how many students are going on the field trip.

$\frac{900x(x+10)}{x} - \frac{900x(x+10)}{x+10} = 15x(x+10)$ LCD = $x(x+10)$ (0.5 mark)

$\frac{900\cancel{x}(x+10)}{\cancel{x}} - \frac{900x\cancel{(x+10)}}{\cancel{x+10}} = 15(x^2+10x)$ (0.5 mark)

$900(x+10) - 900x = 15x^2 + 150x$ (1 mark)

$900x + 9000 - 900x = 15x^2 + 150x$ (1 mark)

$15x^2 + 150x - 9000 = 0$

$15(x^2 + 10x - 600) = 0$ (0.5 mark)

$15(x-20)(x+30) = 0$

$x - 20 = 0; x + 30 = 0$

$x = 20; x \neq -30$

-30 is an extraneous root (Note :deduct 0.5 marks if not stated)

20 students (0.5 mark)